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INTERPRETATION OF A DOUBLE PUMPING TEST AT ASSENEDE (N.W. BELGIUM) BY MEANS OF AN INVERSE NUMERICAL MODEL AND THE COMPARISON OF ORDINARY AND BIWEIGHTED LEAST SQUARE SOLUTIONS

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ABSTRACT. By the execution of a double pumping test the hydraulic parameters of the layered groundwater reservoir under the «Rode Polder» in Assenede (N.W. Belgium) were determined. The accurate knowledge of these hydraulic parameters is very important for the simulation of the evolution of the fresh-salt water distribution under the studied polder area. All drawdowns of both pumping tests are simultaneously interpreted by means of the inverse numerical model. The lithostratigraphical information gathered during the drilling activities is represented accurately in the numerical model which allows the consideration of a large amount of layers. As a result of the inverse model a unique solution is derived where for each hydraulic parameter which can be derived from the observed drawdowns, one value and one marginal standard deviation is obtained. By the application of the biweighted least square method the effect of the outliers on the results is deduced.

KEY WORDS: Pumping test analysis, layered groundwater reservoir, inverse numerical model.

SAMENVATTING. Door de uitvoering van de dubbele pompproef worden de hydraulische parameters afgeleid van het grondwater reservoir onder de Rode Polder te Assenede (N.W. België). De nauwkeurige kennis van deze hydraulische parameter is zeer belangrijk voor de simulatie van de evolutie van de zoet en zout water verdeling onder het bestudeerde poldergebied. Alle verlagingen van beide pompproeven worden gelijktijdig geinterpreteerd door middel van het invers numeriek model. De lithostratigrafische gegevens verzameld tijdens het boren van de pomp- en de waarnemingsputten worden nauwkeurig opgenomen in het numeriek model dat toelaat een groot aantal lagen te meter die afgeleid kan worden uit de waargenomen verlagingen één waarde bekomen wordt en één marginale standard afwijking. Door de toepassing van de methode van de bigewicht kleinste kwadraat afwijking wordt het effect afgeleid van de uitschieters op de resultaten.

SLEUTELWOORDEN: Pompproefanalyse, gelaagd grondwaterreservoir, invers numeriek model.

1. INTRODUCTION

For understanding and/or solving most hydrogeological problems it is important to obtain the accurate knowledge of the horizontal hydraulic conductivities of the pervious layers, the vertical hydraulic conductivities of the semi-pervious layers and in unsteady state cases the specific elastic storage and/or the storage coefficient near the watertable. This knowledge cannot be obtained by the execution of a simple pumping test and the interpretation of the observed drawdowns by the classical interpretation methods based upon analytical models of oversimplified groundwater reservoirs.

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In this paper it is demonstrated how these above mentioned hydraulic parameters can be deduced by the execution of a double pumping test in the polder area of Assenede (East-Flanders). The pumping test was executed in the framework of a fresh-salt groundwater flow study. In those studies of flow with different densities where the vertical components are very important, the accurate knowledge of the vertical hydraulic conductivities of the semi-pervious layers is of prior concern. During the interpretation of the double pumping test all observed drawdowns are simultaneously introduced in the inverse numerical model. This exceeds largely the possibilities of the classical interpretation methods where the observations are fragmented per observation well and into drawdowns and residual drawdowns. The numerical model, which is an axi-symmetric hybrid finite-difference finite-element model (Lebbe, 1988), allows the accurate representation of the lithostratigraphical information collected during the drilling activities of the pumping and observation, wells. The double pumping test is executed in the layered groundwater reservoir formed by tertiary and quaternary deposits situated under the «Zwarte Sluis Polder» near the Dutch-Belgian border. The inverse model is started with the ordinary least square method (OLS- method) and was continued by the biweighted least square method (BWLS-method) to reduce the effect of the outliers. The accuracies are deduced from the variance-covariance matrix of the hydraulic parameters. The inverse model can be considered as a generalized interpretation method of pumping tests.

2. INVERSE MODEL

The inverse model is obtained by a combination of a numerical model with a sensitivity analysis and a non-linear regression analysis. The applied numerical model is two-dimensional axi-symmetric. In the numerical model the groundwater reservoir is subdivided in a number of homogeneous layers which are numbered from bottom to top. Each layer is subdivided in a number of concentric rings. The lowest layer, layer 1, is bounded below by an impervious boundary and the uppermost layer is bounded above by the water table. The horizontal flow and change in storage of each layer are characterized respectively by one value for the horizontal hydraulic conductivity and by one value of the specific elastic storage. The vertical flow between two layers is governed by one value of the hydraulic resistance between the layers. The hydraulic resistance is the thickness of a layer divided by its vertical conductivity. The amount of water delivered by a unit decline of the water table is given by one value of the storage coefficient near the watertable. The drawdowns in the different rings of the different layers at the different time steps are calculated with a hybrid finite-difference finite-element method. During the calculated intervals a linear change is assumed between the drawdown and the logaritm of the time since the start of the pump. A detailed description of the numerical model is given in Lebbe (1988). Also the validation of the numerical model was demonstrated by the simulation of the models of Theis (1935), Jacob (1946), Hantush & Jacob (1955), Hantush (1960,1966) and the model of Boulton (1955,1963) as it was explained by Cooley (1971, 1972) and Cooley & Case (1973).

After the schematization of the groundwater reservoir one has to estimate the initial values of the hydraulic parameters. With these values the model calculates the drawdowns at the same places and times where the observations took place. The differences between the logarithms of the observed and the calculated drawdowns for a certain parameter set are defined as residuals :

or $\mathbf{r} = \log_{10}\mathbf{s}^* - \log_{10}\delta$ (1)

where $\mathbf{r}$ are the residuals,
$\mathbf{s}^*$ the measured drawdowns,
and δ the calculated drawdowns.

To adjust the values of the parameters so that the sum of the squares of the residuals becomes smaller one has to calculate the sensitivities of the drawdowns to the hydraulic parameters or groups of parameters. The ij-th component of the sensitivity matrix or Jacobian matrix is defined as :

$$J_{ij} = (\log_{10}\delta_i(P_j.sf) - \log_{10}\delta_i) / \log_{10}(sf) \qquad (2)$$

where sf is the sensitivity factor,
δ_i is the calculated drawdown at the place and time of the i-th observation with the estimated values of the parameters for the first iteration or calculated values of the foregoing iteration,
and $\delta_i(P_j.sf)$ is the calculated drawdown at the place and time of the i-th observation with the estimated values of the parameters with the exception of the value(s) of the j-th parameter or group of parameters whose estimated value(s) are multiplied with the sensitivity factor.

With the help of the residuals and the sensitivities the adjustment factors are calculated by means of the linearization method (Draper & Smith, 1966).

$$\mathbf{A} = (\mathbf{J}^T\mathbf{wJ})^{-1}\mathbf{J}^T\mathbf{wr} \qquad (3)$$

where $\mathbf{A}$ is the vector of the logarithms of the adjustment factors of the different parameters.
$\mathbf{w}$ is an identity matrix with the same dimension as there are observations if the OLS method is used and a diagonal matrix with the weights of the observations on its diagonal elements if a WLS method is used (for more explanation see below),

The newly estimated values of the parameters are obtained by multiplying the old ones with their corresponding adjustment factors or:

$$P_j^{n+1} = P_j^n \cdot 10^{A_{jn}} \qquad (4)$$

where P_j^n is the value of the j-th parameter during the n-th iteration of the inverse process,
A_j^n is the logarithm in base 10 of the adjustment factor of the j-th parameter deduced after the n-th iteration.

The algorithm is repeated until the adjustment factors become very small and the sum of the squares of the residuals reach a minimum value. In this paper the biweighted least square (BWLS) method as described by Wannacott & Wannacott (1985) is applied. In this method a kind of standardized residual $\mathbf{u}$ is calculated:

$$\mathbf{u} = \mathbf{r} / 3.\mathrm{IQR} \qquad (5)$$

where IQR is the interquartile range.

The weight is now given in a diagonal matrix $\mathbf{w}$ where the weight of the i-th observation is given in the diagonal element w_{ii}:

$$w_{ii} = (1-u_i^2)^2 \text{ if } |u_i| \leq 1 \text{ and } w_{ii} = 0 \text{ if } |u_i| > 1 \qquad (6)$$

When it is assumed that the residuals with their different weights approximate a normal distribution with the mean equal to zero and that the drawdowns can be approximated as a linear function within the considered region then the joint probability distribution can be described by the mean and the variance-covariance matrix of the parameters $\mathbf{cov}_p$:

$$\mathbf{cov}_p = \sigma_s^2\,(\mathbf{J}^T\mathbf{wJ})^{-1} \qquad (7)$$

where σ_s^2 can be estimated as $(\sum^n_{i=1} w_{ii}r_i^2) / (\sum^n_{i=1} w_{ii}-p)$ when n is the number of observations and p the number of parameters.

The marginal standard deviation sm_j of the j-th parameter can now be approximated as the square root of the j-th diagonal term of the variance-covariance matrix. This standard deviation represents the variability when nothing is known about the other parameters.

With the aid of the marginal standard deviation sm_j the N% marginal confidence interval can be approximated. The lower and the upper limits of this confidence interval are obtained by respectively dividing and multiplying the optimal values of the hydraulic parameters with their marginal confidence factors, $CfNm_j$. This marginal confidence factor can be approximated with the following equation:

$$CfNm_j = 10^{(sm_j\sqrt{pF(p,n-p,1-\alpha)})} \qquad (8)$$

where $F(p,n-p,1-\alpha)$ is the F-distribution with p and n-p degrees of freedom and a significance level α (=N/100).

When the estimates are correlated this interval may however be a poor measure of the uncertainty (Carrera & Neuman, 1986).

3. DOUBLE PUMPING TEST

The pumping test side is located in the «Rode polders». This is a part of the «Zwarte Sluis Polder» which is situated just south of the Dutch-Belgian border. The lithostratigraphical cross-section (Fig.1) is based on the description of samples collected during drilling activities and on the results of geophysical borehole loggings (caliper, spontaneous potential, point resistance, natural gamma and resistivity measurements with the long-normal and the short-normal device). In our practical case the natural gamma logging characterizes quite well the layering of the groundwater reservoir (Fig.1).

The base of the considered groundwater reservoir is formed by the heavy clay of the Onderdijke-Adegem Member (a3) with a thickness of about eleven meters. The lowest pervious layer of the groundwater reservoir are the silty fine sands of the Bassevelde Member (s3) with a thickness of 17.5 m. In the middle of the groundwater reservoir a semi-pervious layer occurs which is formed by the sandy clay of the Watervliet Member (zK). The upper part of the considered groundwater reservoir is formed by quaternary sediments. The sediments are separated in three different layers, a medium sand (KZ1), a sandy silt (KL) and a medium to fine sand gradually changing towards the surface to clay. The last mentioned sediments are of Holocene age. Because the watertable is situated near the base of the uppermost pervious layers the saturated part of this layer is rather thin.

The location of the pumping and observation wells and their screen intervals is shown in Figure 1. Two different pumping wells are installed, one in the pervious layer formed in the Bassevelde Member s3 and

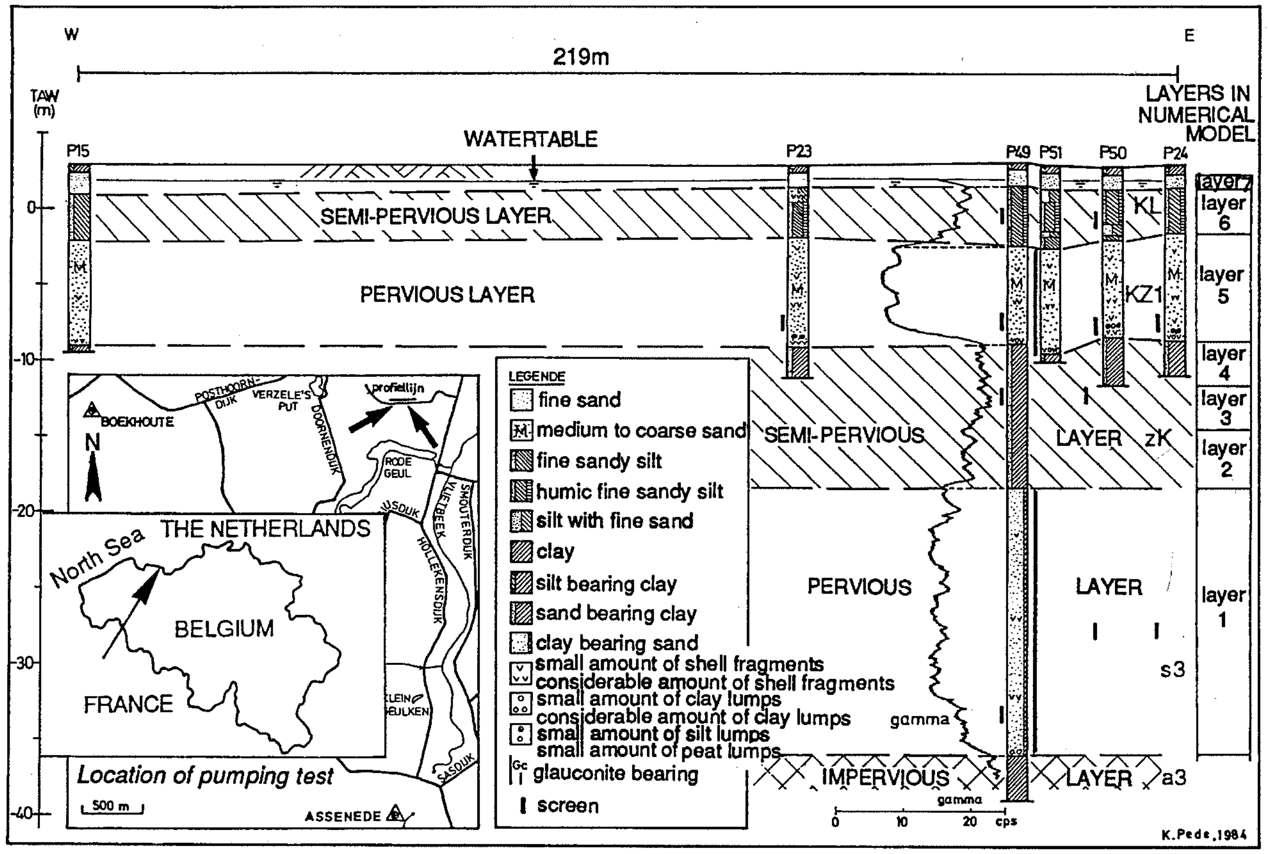

Figure 1. Lithostratigraphical cross-section through the pumping site, location of pumping and observation wells and schematization of the groundwater reservoir in the numerical model.

a second in the pervious layer formed by the lower part of the Pleistocene deposits, KZ1. The observation wells are situated in four different layers: three observation wells are located in the Bassevelde Member s3, two in the Watervliet Member zK, three in the lower part of the Pleistocene KZ1 and finally one in the upper sandy silt of the Pleistocene KL.

During the first pumping test water was extracted from the pumped well which is situated in the lower part of the Pleistocene deposits KZ1. The duration of the test was only one day. One week after the stop of the first pumping test the second pumping test started. During these second pumping test water was extracted from the pumping well in the Bassevelde Member s3. During the two pumping tests water was extracted by means of a submersible pump with the same discharge rate, namely 288 m^3/d.

Schematization of groundwater reservoir in numerical model

In the numerical model the groundwater reservoir is schematized in seven layers. Layer 1 corresponds with the fine sands of the Bassevelde Member s3. The sandy clay of the Watervliet Member zK is subdivided in three layers in the numerical model. This subdivision in the numerical model was necessary to simulate accurately the drawdown in the middle of the sandy clay during the two pumping test. Layer 5 of the numerical model corresponds with the medium fine sands KZ1. Layer 6 of the numerical model is the sandy silt KL. The uppermost layer of the numerical model, layer 7, is the very thin sandy layer just underneath the watertable.

Hydraulic parameters derived from the observed drawdowns

Studying the sensitivities and the variance-covariance matrices generated with the initial estimates of the parameter values the following hydraulic parameters can be deduced by means of the inverse model from the observed and residual drawdowns of the two pumping tests. The horizontal hydraulic conductivities of the pumped pervious layers 1 and 5, $k^h(1)$ and $k^h(5)$, can be deduced separately. The horizontal hydraulic conductivities of the other layers 2, 3, 4, 6 and 7 are unidentifiable. So introducing rough estimates of their values was sufficient. These values do

not have a significant influence on the values deduced for the other parameters. Because layers 2, 3 and 4 of the numerical model represents the same lithostratigraphical layer a same value of 0.2 m/d was attributed to the horizontal conductivity of these layers. The horizontal hydraulic conductivity of layer 6, representing the silty deposits KL, is estimated at 0.02 m/d. The horizontal hydraulic conductivities of the uppermost layer of the inverse model is set equal to 2.5 m/d.

The hydraulic resistances between the different layers are grouped in for different groups of identifiable parameters. The hydraulic resistances between layers 1 and 2, c(1), and between layers 2 and 3, c(2) are identifiable as a group. Their sum is equal to the hydraulic resistance between the top of layer s3 and the screen of the observation wells situated in the sandy clay zK. The hydraulic resistances c(3) and c(4) are considered together. The sum of their values is equal to the hydraulic resistance of the upper part of the sandy clay zK. The hydraulic resistances c(5) and c(6) are considered to be separately identifable. They correspond with the hydraulic resistance between the base of the screen of the observation well in the sandy silt KL and with the hydraulic resistance between the top of the screen of the last mentioned observation well and the watertable.

The specific elastic storages of the lowermost pumped layer 1, Ss(1), is separately identifiable. The specific elastic sto-rages Ss(2), Ss(3), Ss(4) are included in one group and it is assumed that they have the same value. The last group of parameters which are identifiable are the specific elastic storages Ss(5), Ss(6), Ss(7). The storage coefficient near the watertable S0 was not identifiable and was set equal to the estimated value 0.125.

Interpretation of the results

The results of the OLS-method are represented in Table 1.

In this table the optimal values of the hydraulic parameters are given together with their marginal standard deviations, sm_j, and the marginal confidence factor, $Cf98m_j$. From these last two statistical parameters one can deduce the accuracy with which the values can be deduced from the observations. So it is clear that the obtained value for the hydraulic resistance c(6) can not be considered as been deduced. His marginal standard deviation is too large. This is because of the small sensitivities of the calculated drawdowns for this hydraulic parameter corresponding with the time and the place of the observations and also because of the large residuals of observations which are the most sensitive to these hydraulic parameters.

The horizontal conductivities of the pumped layer $k^h(1)$ and $k^h(5)$, the specific elastic storage of the layer $S_s(1)$ and the hydraulic resistance of the lower part of the sandy clay zK are rather well determined while the accuracies of the other para-meters are rather weak.

The large marginal standard deviations of the different hydraulic parameters are caused by the rather high sum of the squares of the residuals which are still there after the optimalization. In Fig. 2 the calculated and measured drawdowns of the double pumping test are represented for the different layers. From this figure one can see that the largest residuals corresponds with the observation in the non-pumped pervious layers during the double pumping test.

By the application of the biweighted least square method (BWLS method) the influence of the largest

Parameter	Unit	Value	sm_j	$Cf98m_j$
$k^h(1)$	m/d	1.37	0.0221	1.221
$k^h(5)$	m/d	17.2	0.0243	1.246
c(1,2)	d	3990	0.0312	1.326
c(3,4)	d	58.5	0.0574	1.681
c(5)	d	76.0	0.0892	2.242
c(6)	d	4438	0.4029	38.34
$S^s(1)$	m^{-1}	$4.9\ 10^{-5}$	0.0284	1.293
$S^s(2\text{-}4)$	m^{-1}	$4.0\ 10^{-6}$	0.0806	2.074
$S^s(5\text{-}7)$	m^{-1}	$4.9\ 10^{-5}$	0.0635	1.777

Table 1. Optimal parameter values obtained by the ordinary least square (OLS-) method.

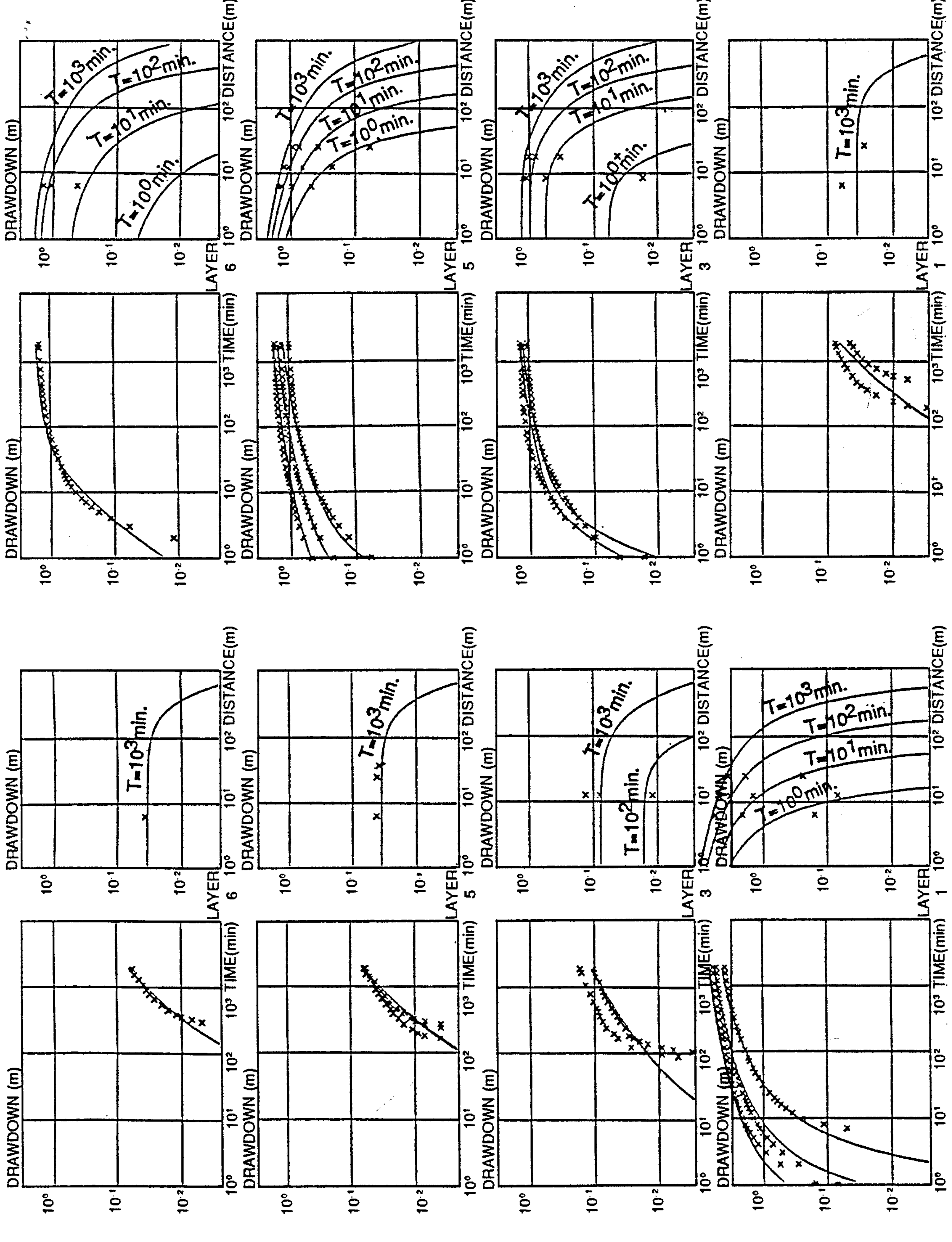

Figure 2. Measured (x-signs) and calculated (solid curves) draw-downs in time- and distance-drawdown graphs for the two pumping tests (left two columns for pumping test in layer s3 and right two columns for the pumping test in layer KZ1, discharge rate in both pumping test is equal to 288 m^3/d). Calculated drawdown with the optimal values of the OLS solution.

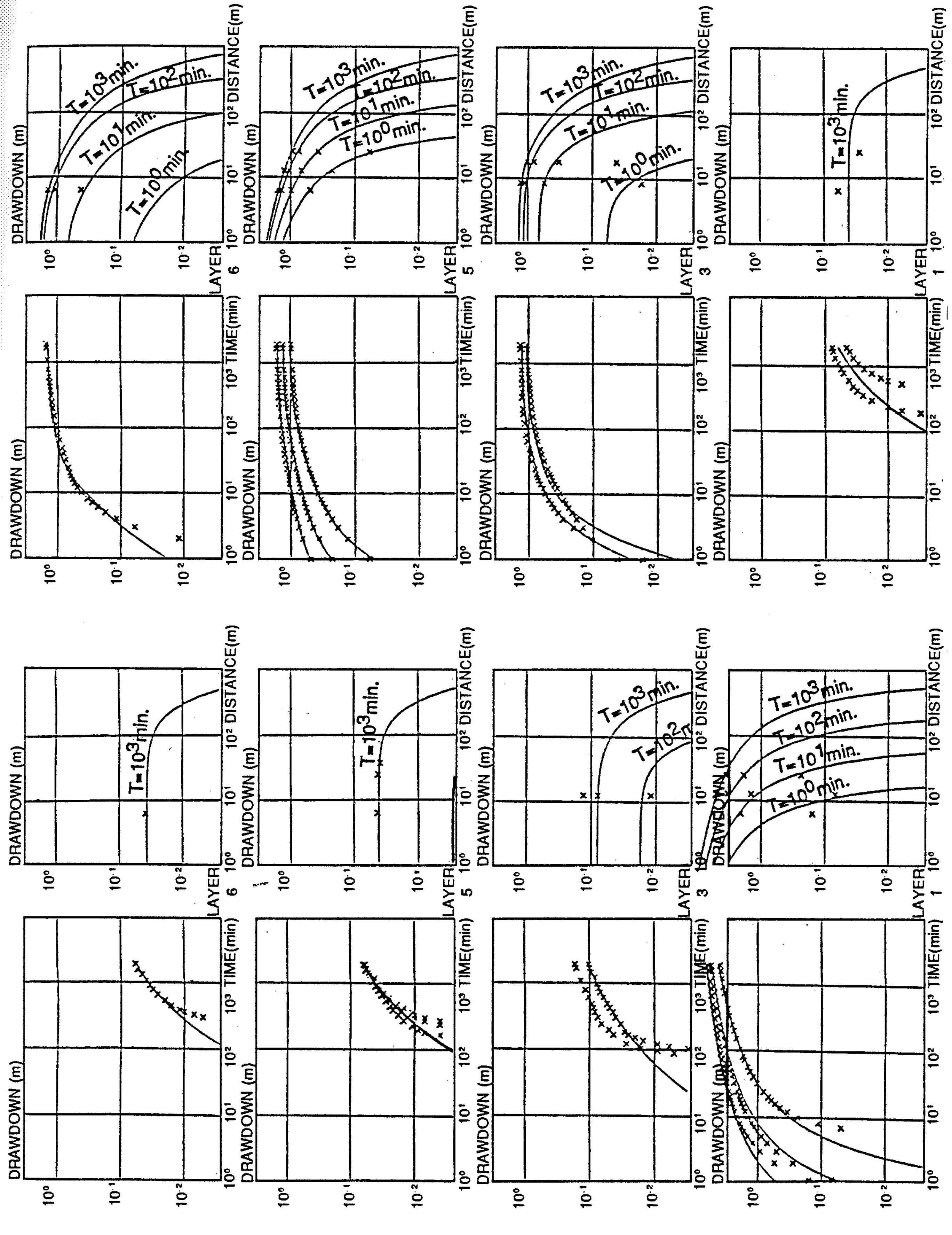

Figure 3. Measured (x-signs) and calculated (solid curves) draw-downs in time- and distance-drawdown graphs for the two pumping tests (left two columns for pumping test in layer s3 and right two columns for the pumping test in layer KZ1, discharge rate in both pumping test is equal to 288 m^3/d). Calculated drawdown with the optimal values of the BWLS solution.

residuals or the outliers on the results are deduced. In Table 2 the evolution of the values of the hydraulic parameters after each iteration of the inverse model is given. After each iteration the weights attributed to each observation change and so their sum. The evolution of the total weight is also represented in Table 2.

In Fig. 3 the calculated and the measured drawdowns of the double pumping test are represented corresponding with the parameter values obtained with the BWLS method. Although the change in the calculated drawdown is rather small the values of some hydraulic parameters change considerably. This is in particular the case with the values of the hydraulic parameters which has the three largest values for their marginal standard deviations in the OLS solution (Table 1). The hydraulic resistance c(6) changes from 4438 d to 278 d, the specific elastic storage of layer zK, $s_s(2\text{-}4)$, changes from 4.0 10^{-6} m^{-1} to the more acceptable value 1.1 10^{-5} m^{-1}, and the hydraulic resistance c(5) changes from 76 d to 44 d. The other parameter values corresponding with a smaller marginal standard deviation in the OLS solution are not considerablely different from those obtained with the BWLS method. Because of the elimination of the outliers in the BWLS method and the attribution of small weights to the observations with rather high residuals the sum of the weighted squares of the residuals, $\Sigma w_{ii}.r_i^2$, is much smaller than in the OLS method where each w_{ii} is equal to one. This meaningfull smaller sum results in much smaller marginal standard deviations for the different hydraulic parameters for the BWLS solution.

As a result of the pumping test and the interpretation with the OLS- and the BWLS-method one can conclude that the horizontal conductivities of the two pervious layers are deduced with the highest accuracy. The horizontal conductivity of the layer s3 is 1.43 m/d and the horizontal conductivity of layer KZ1 is 14.3 m/d. Three hydraulic parameters are deduced with a rather high accuracy. They are the specific elastic storage of the pervious layer s3 and KZ1 which are respectively $4.1.10^{-5}$ and $5.6.10^{-5}$ m^{-1} and the hydraulic resistance of the lower half of the semi-pervious layer zK, 4700 d which results in a vertical hydraulic conductivity of $1.1.10^{-3}$ m/d.

4. CONCLUSION

The inverse numerical model allows the interpretation of all observed drawdowns simultaneously. The fragmentation of the observation in time-drawdown curves or in distance drawdown curves to compare with some type-curves is no longer necessary. By the numerical model it becomes possible to schematize

Iteration	Total weight	$k^h(1)$ m/d	$k^h(5)$ m/d	c(1,2) d	c(3,4) d	c(5) d	c(6) d	$S_s(1)$ m^{-1}	$S_s(2\text{-}4)$ m^{-1}	$S_s(5\text{-}7)$ m^{-1}
1	320.	1.43	16.2	7148.	110.	63.1	413.	$4.8.10^{-5}$	$3.5.10^{-6}$	$5.0.10^{-5}$
2	314.	1.46	15.5	6216.	86.8	51.9	578.	$4.4.10^{-5}$	$5.5.10^{-6}$	$5.5.10^{-5}$
3	306.	1.48	15.1	5636.	74.3	48.6	445.	$4.4.10^{-5}$	$7.7.10^{-6}$	$5.4.10^{-5}$
4	301.	1.49	14.8	5188.	64.2	47.2	366.	$4.3.10^{-5}$	$9.2.10^{-6}$	$5.3.10^{-5}$
5	301.	1.50	14.6	4748.	60.4	45.7	316.	$4.2.10^{-5}$	$1.0.10^{-5}$	$5.4.10^{-5}$
6	301.	1.51	14.4	4748.	58.6	44.5	290.	$4.2.10^{-5}$	$1.1.10^{-5}$	$5.5.10^{-5}$
7	301.	1.52	14.3	4716.	56.6	43.8	278.	$4.1.10^{-5}$	$1.1.10^{-5}$	$5.6.10^{-5}$
8	301.	1.52	14.3	4716.	56.4	43.5	278.	$4.1.10^{-5}$	$1.1.10^{-5}$	$5.6.10^{-5}$
sm_j	-	.0049	.0045	.0091	.0158	.0148	0.245	.0090	.0222	.0101
$Cf98m_j$	-	1.045	1.045	1.086	1.154	1.143	1.248	1.086	1.223	1.096

Table 2. Evolution of parameter values and the total weight during the iterations of the biweighted least square (BWLS-) method.

accurately the groundwater reservoir. The lithostratigraphical information gathered from drilling and geophysical logs can be used in an optimal way. The inverse model allows not only the interpretation of observed drawdowns in the pumped layer but also the drawdowns measured in the layers adjacent to the pumped layer. In contrast with the classical interpretation methods, consisting of fitting different observed drawdown curves and resulting in a series of different values for each hydraulic parameter, the inverse model gives a unique solution where each hydraulic parameter obtains one value and one marginal standard deviation. This last mentioned statistical parameter can be considered as a measure for the accuracy of the deduced hydraulic parameters. By eliminating the outliers and applying the BWLS-method a better idea about the accurancy of the deduced parameters can be obtained. Those parameters which have a large difference between their estimates obtained by the OLS-method and with the BWLS-method are the parameters with the largest marginal standard deviation in the OLS-solution.

With the double pumping test in the «Rode Polders» the horizontal hydraulic conductivities and the specific elastic storage of the pumped layers are well determined together with the vertical hydraulic conductivity of the semi-pervious layer between these two pumped layers.

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6. NOTATIONS

- **A** vector of logarithm of adjustment factors according to the different parameters, order p.
- $\mathbf{cov_p}$ covariance matrix of the parameters, order pxp.
- $CfNm_j$ marginal confidence factor which determines the upper and lower limits of the N% marginal confidence interval of the j-th parameter.
- c(l) hydraulic resistance between the middles of the layers l and l+1 of the numerical model $=D(l)/2*k_v(l)+D(l+1)/2*k_v(l+1)$, (T).
- D(l) thickness of layer l of the numerical model, (L).
- **J** sensitivity or Jacobian matrix, order nxp.
- $k_h(l)$ horizontal hydraulic conductivity of the layer l of the numerical model, (LT^{-1}).
- $k_v(l)$ vertical conductivity of the layer l of the numerical model, (LT^{-1}).
- n number of observed drawdowns.
- p number of deduced parameters.
- **P** vector of the parameter values, order p.
- Q(l) discharge rate pumped in layer l of numerical model, L^3T^{-1}.
- **r** vector of residuals, order n.
- δ vector of calculated drawdowns, order n.
- **s*** vector of measured drawdowns, order n.
- sf sensitivity factor, dimensionless.
- sm_j marginal standard deviation of j-th parameter in logaritmic region.
- Ss(l) specific elastic storage of layer l of the numerical model, (L^{-1}).
- S0 storage coefficient near the water table also called specific yield, (L^3L^{-3}).
- **u** Vector of standardized residuals, order n.
- **w** diagonal matrix of weights, order nxn.
- α level of significance.
- Σ summation sign.

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