ROYAUME DE BELGIQUE

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CALCUL DES ECOULEMENTS DE NAPPES AQUIFERES A DEUX DIMENSIONS, EN REGIME TRANSITOIRE

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PROFESSIONAL PAPER 1986/4
N° 227

Service Géologique de Belgique - Professional Paper n° 227 1986 / 4

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En témoignage de gratitude aux fidèles amis qui m'ont donné l'occasion d'utiliser une H.P. 41, calculette sans laquelle ce travail n'aurait pas vu le jour.

PREFACE

L'application de la méthode pseudovariationnelle de Maurice A. Biot à l'étude des nappes aquifères confinées [2] s'est révélée puissante pour résoudre les problèmes classiques de l'hydrologie souterraine. La monographie que nous présentons envisage de nombreux cas où des solutions analytiques sont connues et tabulées, cependant, quelques résultats sont nouveaux. C'est ainsi que nous donnons la solution du problème de Boussinesq (1903) pour les nappes libres aussi bien en écoulement parallèle qu'en écoulement axisymétrique. Etant donné la signification physique différente du coefficient d'emmagasinement ment S et de la porosité effective, f, il n'est pas possible de comparer les solutions obtenues pour les nappes libres et pour les nappes confinées. Cependant la différence d'allures des courbes est significative.

Notre but est essentiellement pratique, aussi ne faisons-nous pas appel à l'équation de Lagrange, encore que celle-ci se déduise le plus naturellement du monde par une intégration par partie de la relation de Darcy mise sous une forme variationnelle.

Les seules relations nécessaires sont, d'une part, la loi de continuité et d'autre part, la loi de Darcy. Tandis que la première de ces relations reste intacte tout au long des calculs, laseconde - loi expérimentale de Darcy - est approximée.

Si le travail n'était fastidieux, on retrouverait facilement par cette méthode tous les résultats que Carslaw and Jaeger ont accumulés dans leur classique "Conduction of heat in solids" (1959), avec cet avantage qu'on peut se passer de toutes les fonctions transcendantes et résoudre les équations différentielles par une calculette de poche munies de mémoires supplémentaires (H.P. 41).

Ce travail se voudrait un hommage à Maurice A. Biot en montrant l'efficacité de sa méthode dans la solution de problèmes non linéaires ou de problèmes dont les solutions analytiques sont difficilement utilisables en raison de leur complexité.

Les résultats relatifs aux nappes confinées sont repris afin de faciliter les comparaisons avec ceux que nous établissons pour les nappes libres et pour uniformiser les notations.

Cependant on n'oubliera pas la différence physique fondamentale entre un écoulement strictement radial en nappe confinée et celui à trois dimensions des nappes libres où l'hypothèse Dupuit-Forshheimer doit certaienement couduire à une approximation bien moins bonne qu'en nappe confinée.

AVERTISSEMENT

1. Les calculs algébriques nécessités par la méthode sont élémentaires même s'ils sont quelquefois un peu longs. Une lettre pointée désigne comme à l'habitude une dérivation par rapport au temps. Ainsi si $\dot{Q}$ désigne un débit, la quantité d'eau Q est définie par $\int \dot{Q}\,dt$

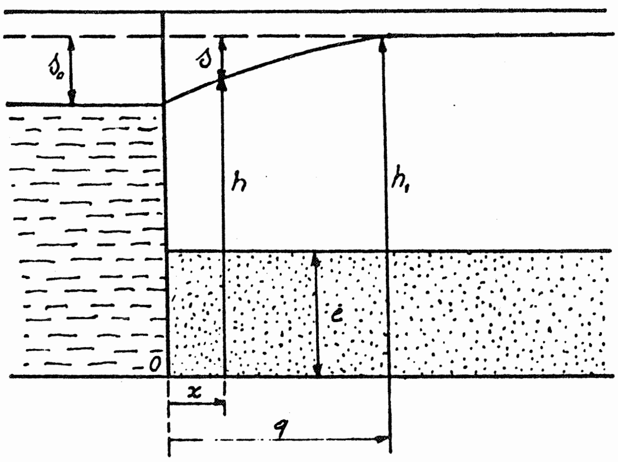
2. q en écoulement parallèle et R en écoulement axisymétrique désignent la distance ou le rayon d'influence (profondeur de pénétration). L'allure choisie pour les courbes de rabattement : $s = s(q \text{ ou } R)$ correspond à ce qu'elle est en régime permanent et n'est donc pas tout à fait arbitraire.

3. D'aucuns estimeront puérils la production de tableaux de chiffres et de courbes tracées par point à la main alors que des imprimantes et tables traçantes associées aux calculatrices donnent ces documents bien plus rapidement. D'autant plus que ces moyens automatiques permettent le tracé de familles nombreuses de courbes pour autant de valeurs du paramètre qu'on le souhaite et cela avec une grande précision. Ceci est parfaitement exact mais, outre le fait de ne pouvoir disposer de ces outils, j'ai voulu prouver la facilité avec laquelle cette méthode peut se mettre en oeuvre avec des moyens limités.

Lorsque des formules analytiques existent, nous avons comparé les résultats qu'elles fournissent avec ceux auxquels la présente méthode aboutit. La concordance est si bonne qu'on en viendrait à imaginer une méthode d'approximation des fonctions transcendantes en les considérant comme solution d'équations différentielles du premier ordre.

SOMMAIRE

1.1.1. Nappe confinée. Ecoulement parallèle. Soutirage à niveau constant s_0.



Solution analytique connue :

(Ingersoll, Zobel et Ingersoll, 1948)

$$s = s_0\,\mathrm{erfc}(u) = s_0\,D(u) \quad \text{avec} \quad u^2 = \frac{S x^2}{4Tt}$$

$$\dot Q_0 = s_0\sqrt{\frac{ST}{\pi t}}$$

$\dot Q_0$ débit en provenance d'un seul côté.

Méthode pseudo variationnelle.

Continuité : $\frac{d\dot Q}{dx} = -S\dot h$ ou $\frac{d\dot Q}{dx} = S\dot s$

Darcy : $\dot Q + T\frac{dh}{dx} = 0$ ou $\int_0^q \dot Q\,\delta Q\,dx = T\int_0^q \frac{ds}{dx}\,\delta Q\,dx$ avec $\begin{cases}\delta Q = \frac{\partial Q}{\partial q}\delta q \\ \dot Q = \frac{\partial Q}{\partial q}\dot q\end{cases}$

d'où $$\dot q\int_0^q\left(\frac{\partial Q}{\partial q}\right)^2 dx = T\int_0^q \frac{ds}{dx}\frac{\partial Q}{\partial q}\,dx \qquad (1)$$

Soit une allure plausible pour s : $s = s_0\zeta^2$ avec $\zeta = 1-\frac{x}{q}$

$\frac{d\zeta}{dx} = -\frac{1}{q}$; $\frac{ds}{dx} = 2s_0\zeta\frac{d\zeta}{dx} = -2s_0\frac{\zeta}{q}$; $dx = -q\,d\zeta$ et $\frac{d\zeta}{dq} = \frac{x}{q^2} = \frac{1-\zeta}{q}$ et (1) devient :

$$q\dot q\int_0^1\left(\frac{\partial Q}{\partial q}\right)^2 d\zeta = -2s_0T\int_0^1 \zeta\,\frac{\partial Q}{\partial q}\,d\zeta \qquad (2)$$

or $Q = -S\int_x^q s\,dx = -Sqs_0\int_0^\zeta \zeta^2 d\zeta = -\frac{1}{3}Sqs_0\zeta^3$ et $Q_0 = -\frac{1}{3}qs_0S$

$\frac{\partial Q}{\partial q} = -\frac{Ss_0}{3}\left[\zeta^3 + 3q\zeta^2\frac{\partial\zeta}{\partial q}\right] = -\frac{Ss_0}{3}\zeta^2(3-2\zeta)$ et (2) devient:

$q\dot q\frac{S}{3}\int_0^1\zeta^4(3-2\zeta)^2 d\zeta = 2T\int_0^1\zeta^3(3-2\zeta)\,d\zeta$ ou $q\dot q\left(\frac{9}{5}-\frac{12}{6}+\frac{4}{7}\right) = \frac{6T}{S}\left(\frac{3}{4}-\frac{2}{5}\right)$ ou

$q\dot q = \frac{147}{26}\frac{T}{S}$ d'où $q^2 = \frac{147}{13}\frac{Tt}{S}$ et $s = s_0\left(1-\frac{x}{q}\right)^2 = s_0\left(1-\frac{2x}{\sqrt{\frac{147}{13}\frac{4Tt}{S}}}\right)^2$ ou, en posant :

$u^2 = \frac{Sx^2}{4Tt}$ $\boxed{s = s_0\left(1-2\sqrt{\frac{13}{147}}\,u\right)^2 = s_0\,D'(u)}$

$\dot Q_0 = -\frac{Ss_0}{3}\dot q = \frac{-Ss_0}{3}\sqrt{\frac{147}{13}\frac{T}{S}}\frac{1}{2\sqrt t}$ ou $\boxed{\dot Q_0 = -s_0\sqrt{\frac{ST}{\frac{156}{49}t}}}$

Comparaison entre $D(u) = \mathrm{erfc}(u)$ et $D'(u) = \left(1 - 2\sqrt{\frac{13}{147}}\,u\right)^2$

u	$D(u)$	$D'(u)$	u	$D(u)$	$D'(u)$
0.03162	0.9643	0.9627	0.3162	0.6548	0.6592
0.04	0.9549	0.9529	0.4	0.5716	0.5807
0.05	0.9436	0.9414	0.5	0.4795	0.4936
0.06325	0.9287	0.9262	0.6325	0.3711	0.3891
0.07746	0.9128	0.9099	0.7746	0.2733	0.2908
0.08944	0.8994	0.8964	0.8944	0.2059	0.2190
0.1	0.8875	0.8845	1	0.1573	0.1642
0.1265	0.8580	0.8551	1.14	0.1069	0.1036
0.1581	0.8231	0.8207	1.265	0.0736	0.0613
0.2	0.7730	0.7762	1.378	0.0513	0.0325
0.2449	0.7291	0.7299	1.483	0.0359	0.0139
0.2828	0.6892	0.6918	1.581	0.0254	0.0035

Pour $u = \sqrt{\frac{147}{52}} = 1.6813..$ $D'(u) = 0$

Le problème : $T\frac{\partial^2 s}{\partial x^2} = S\frac{\partial s}{\partial t}$ avec $s(0,t) = s_0$, $s(x,0) = 0$, $s(\infty,t) = 0$

On demande $s = s(x,t)$ et $\dot{Q}_0 = T\left(\frac{\partial s}{\partial x}\right)_{x=0}$?

Solution analytique.

$$s = s_0 D(u) = s_0\left[1 - \frac{2}{\sqrt{\pi}}\int_0^u e^{-u^2}du\right]$$

Solution par la méthode M. Biot.

$$s = s_0 D'(u) = s_0\left(1 - \sqrt{\frac{52}{147}}\,u\right)^2$$

avec $u^2 = \frac{Sx^2}{4Tt}$

$$\dot{Q}_0 = s_0\sqrt{\frac{ST}{\pi t}}$$

$$\dot{Q}_0 = s_0\sqrt{\frac{ST}{\frac{156}{49}t}}$$

$\frac{159}{49} = 3.1836$

$\dot{Q}_0$ est le débit en provenance d'un seul côté.

1.1.2. Nappe confinée. Ecoulement parallèle. Soutirage à débit constant : $-\dot{Q}_0$ (Débit en provenance d'un seul côté).

Solution analytique connue (Theis, 1938; Ferris, 1950)

$$s = \frac{-\dot{Q}_0 x}{T}\left[\frac{e^{-u^2}}{u\sqrt{\pi}} - 1 + \frac{2}{\sqrt{\pi}}\int_0^u e^{-u^2}du\right] = -\frac{\dot{Q}_0 x}{T}\left[\frac{e^{-u^2}}{u\sqrt{\pi}} - erfc(u)\right] \quad \text{avec} \quad u^2 = \frac{Sx^2}{4Tt}$$

$$s_0 = \frac{-2\dot{Q}_0\sqrt{t}}{\sqrt{TS\pi}}$$

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continuité : $\frac{d\dot{Q}}{dx} = -S\dot{h}$ ou $\frac{dQ}{dx} = Ss$

Darcy : $\dot{Q} = -T\frac{dh}{dx}$ ou $\dot{Q} = T\frac{ds}{dx}$ d'où $\int_0^q \dot{Q}\,\delta Q\,dx = T\int_0^q \frac{ds}{dx}\,\delta Q\,dx$

avec $\begin{cases} \delta Q = \frac{\partial Q}{\partial q}\delta q + \frac{\partial Q}{\partial s_0}\delta s_0 \\ \dot{Q} = \frac{\partial Q}{\partial q}\dot{q} + \frac{\partial Q}{\partial s_0}\dot{s}_0 \end{cases}$ d'où $\int_0^q \left(\frac{\partial Q}{\partial q}\dot{q} + \frac{\partial Q}{\partial s_0}\dot{s}_0\right)\frac{\partial Q}{\partial q}dx = T\int_0^q \frac{ds}{dx}\frac{\partial Q}{\partial q}dx \quad (')$

Soit $s = s_0\zeta^2$ avec $\zeta = 1 - \frac{x}{q}$ $\quad \begin{cases} dx = -q\,d\zeta & ; \frac{ds}{dx} = -\frac{2s_0\zeta}{q} \\ \frac{d\zeta}{dx} = -\frac{1}{q} & ; \frac{d\zeta}{dq} = \frac{1-\zeta}{q} \end{cases}$

(') devient : $q\int_0^1 \left(\frac{\partial Q}{\partial q}\dot{q} + \frac{\partial Q}{\partial s_0}\dot{s}_0\right)\frac{\partial Q}{\partial q}d\zeta = -2s_0T\int_0^1 \zeta\frac{\partial Q}{\partial q}d\zeta \quad (^2)$

$$Q = -S\int_x^q s\,dx = -Sq s_0\int_0^\zeta \zeta^2 d\zeta = -Sqs_0\frac{\zeta^3}{3} \quad \therefore Q_0 = -\frac{Sqs_0}{3}$$

$\frac{\partial Q}{\partial q} = -S\frac{s_0}{3}\zeta^2(\zeta + 3 - 3\zeta) = -\frac{Ss_0\zeta^2}{3}(3-2\zeta)$ et $\frac{\partial Q}{\partial s_0} = -\frac{Sq\zeta^3}{3}$

$\left(\frac{\partial Q}{\partial q}\right)^2 = \frac{S^2s_0^2}{9}\zeta^4(3-2\zeta)^2$ et $\left(\frac{\partial Q}{\partial q}\right)\left(\frac{\partial Q}{\partial s_0}\right) = \frac{S^2qs_0}{9}\zeta^5(3-2\zeta)$ et $(^2)$ devient :

$$q\dot{q}\frac{S^2s_0^2}{9}\int_0^1 \zeta^4(3-2\zeta)^2d\zeta + q\dot{s}_0\frac{S^2qs_0}{9}\int_0^1 \zeta^5(3-2\zeta)d\zeta = 2s_0TS\frac{s_0}{3}\int_0^1 \zeta^3(3-2\zeta)d\zeta \quad \text{ou}$$

$$\frac{S^2s_0^2}{9}q\dot{q}\underbrace{\left[\frac{9}{5} - \frac{12}{6} + \frac{4}{7}\right]}_{13/35} + \frac{1}{9}S^2q^2s_0\dot{s}_0\underbrace{\left[\frac{3}{6} - \frac{2}{7}\right]}_{3/14} = \frac{2}{3}s_0^2TS\underbrace{\left(\frac{3}{4} - \frac{2}{5}\right)}_{7/20} \quad \text{ou}$$

$\frac{13}{105}\frac{\dot{q}}{q} + \frac{1}{14}\frac{\dot{s}_0}{s_0} = \frac{7}{10}\frac{T}{Sq^2} \quad (^3)$. La contrainte $\dot{Q}_0$ = constante s'écrit : $\dot{Q}_0 t = Q_0$ et devient $\frac{\dot{q}}{q} + \frac{\dot{s}_0}{s_0} = \frac{1}{t}$ et $(^3)$ devient : $\frac{11}{105}\frac{\dot{q}}{q} + \frac{1}{7t} = \frac{7}{5}\frac{T}{Sq^2}$ dont la solution est :

$q = \sqrt{\frac{294}{41}\frac{Tt}{S}}$ d'où :

$$s_0 = \frac{-3\dot{Q}_0t}{Sq} = \frac{-2\dot{Q}_0}{\underbrace{\sqrt{\frac{4\times 294}{369}}}_{3.1869\sim\pi}}\sqrt{\frac{t}{ST}} \quad \text{et} \quad s = s_0\left(1 - \frac{x}{q}\right)^2$$

ou, en posant $u^2 = \frac{Sx^2}{4Tt}$ $\quad s = \frac{-\dot{Q}_o}{T} \frac{x}{\sqrt{\frac{392}{123}}} u\left(\frac{1}{u} - \sqrt{\frac{82}{147}}\right)^2$

Le problème $T\frac{\partial^2 s}{\partial x^2} = S\frac{\partial s}{\partial t}$ avec : $s(\infty,t)=0$; $s(x,0)=0$; $T\left(\frac{\partial s}{\partial x}\right)_{x=0} = \dot{Q}_o$

<u>Solution analytique</u> (Ferris, 1950)

$$\frac{s}{\left(-\frac{\dot{Q}_o}{T}\right)x} = \frac{e^{-u^2}}{u\sqrt{\pi}} - 1 + \frac{2}{\sqrt{\pi}}\int_0^u e^{-u^2}du = D(u)$$

$$s_o = \frac{-2\dot{Q}_o\sqrt{t}}{\sqrt{TS\pi}}$$

<u>Solution méthode Biot.</u>

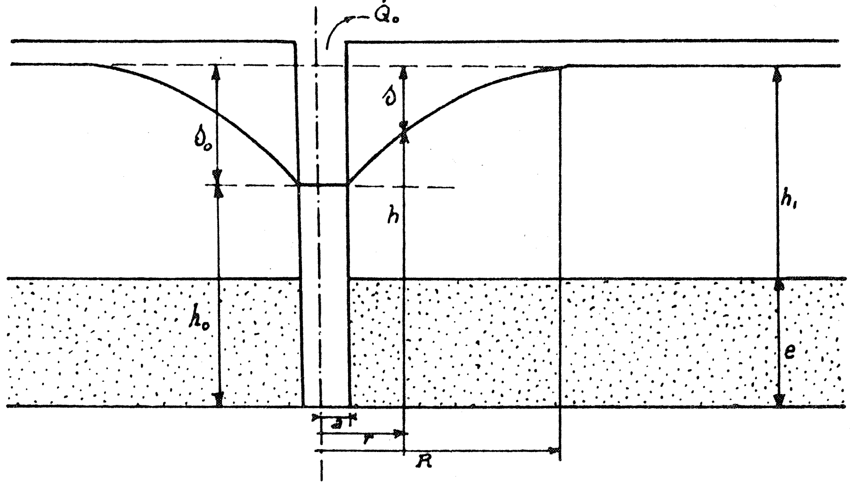
$$\dot{q} = -\frac{15}{11}\frac{q}{t} + \frac{147}{11}\frac{T}{Sq}$$

$$\frac{s}{\left(-\frac{\dot{Q}_o}{T}\right)x} = u\left(\frac{1}{u} - \sqrt{\frac{82}{147}}\right)^2 \frac{1}{\sqrt{\frac{392}{123}}} = D'(u)$$

$$u^2 = \frac{Sx^2}{4Tt} \qquad s_o = \frac{-2\dot{Q}_o}{\sqrt{\frac{392}{123}} = 3.1869}\sqrt{\frac{t}{ST}}$$

u	$D(u)$	$D'(u)$
0.003	187.0649	185.8831
0.004	140.0496	139.2037
0.006	93.03498	92.5246
0.008	69.5282	69.1853
0.01	55.4246	55.1820
0.02	27.2207	27.1773
0.04	13.1273	13.1797
0.06	8.43699	8.5179
0.08	6.0974	6.1902
0.1	4.6982	4.7961
0.2	1.9330	2.0265
0.4	0.6303	0.6886
0.6	0.2599	0.2843
0.8	0.1139	0.1134
1	0.0502	0.0359
1.1	0.03375	0.0162
1.2	0.02170	0.005
1.3389	0.01170	0

1.2.1. Nappe confinée. Ecoulement axisymétrique. Soutirage à niveau constant : s_0.



Solution analytique (Jacob et Lohman, 1952)

$$\dot{Q}_0 = -2\pi T s_0 \, G(\alpha) \quad \text{avec} \quad \alpha = \frac{Tt}{Sa^2} \quad \text{et}$$

$$G(\alpha) = \frac{4\alpha}{\pi}\int_0^{\infty} x\, e^{-\alpha x^2}\left[\frac{\pi}{2} + tg^{-1}\frac{Y_0(x)}{J_0(x)}\right]dx \quad \text{ou, sous une autre forme}$$

$$= \frac{4}{\pi^2}\int_0^{\infty} e^{-\alpha x^2}\frac{dx}{x\left[J_0^2(x) + Y_0^2(x)\right]}$$

$J_0(x)$: fonction de Bessel, d'ordre 0 et de 1ère espèce.
$Y_0(x)$: " " " " " " " 2ème " .

Méthode pseudo-variationnelle.

Continuité : $\dfrac{d\dot{Q}}{dr} = -2\pi r S \dot{s}$ ou $\dfrac{dQ}{dr} = 2\pi r S s$

Darcy : $\dot{Q} = 2\pi r T \dfrac{ds}{dr}$ ou $\displaystyle\int_a^R \dot{Q}\,\delta Q \frac{dr}{r} = 2\pi T \int_a^R \frac{ds}{dr}\,\delta Q\, dr$ avec $\begin{cases} \dot{Q} = \dfrac{\partial Q}{\partial R}\dot{R} \\ \delta Q = \dfrac{\partial Q}{\partial R}\delta R \end{cases}$

$$\text{d'où} \quad \dot{R}\int_a^R \left(\frac{\partial Q}{\partial R}\right)^2 \frac{dr}{r} = 2\pi T \int_a^R \frac{ds}{dr}\frac{\partial Q}{\partial R}\, dr \qquad (')$$

Soit $s = s_0 \dfrac{u}{u_0}$ avec $\begin{cases} u = -\log\dfrac{r}{R} \text{ d'où } \dfrac{r}{R} = e^{-u} \; ; dr = -R e^{-u} du = -a\dfrac{e^{-u}}{e^{-u_0}} du \\ u_0 = -\log\dfrac{a}{R} \quad \dfrac{a}{R} = e^{-u_0} \; ; \dfrac{dr}{r} = -du \; ; r\,dr = a^2\dfrac{e^{-2u}}{e^{-2u_0}} du \end{cases}$

(') devient : $\displaystyle -\dot{R}\int_{u_0}^{0}\left(\frac{\partial Q}{\partial R}\right)^2 du = -2\pi T \frac{a}{e^{-u_0}}\int_{u_0}^{0}\frac{ds}{dr}\left(\frac{\partial Q}{\partial R}\right)e^{-u} du$ ou, puisque :

$$\dot{R} = a e^{u_0}\dot{u}_0 \; ; \; \frac{\partial Q}{\partial R} = \frac{e^{-u_0}}{a}\frac{\partial Q}{\partial u_0} \; ; \; \frac{ds}{dr} = -\frac{e^{-u_0}}{a e^{-u}}\frac{s_0}{u_0}$$

$$\dot{u}_0\int_0^{u_0}\left(\frac{\partial Q}{\partial u_0}\right)^2 du + 2\pi T\frac{s_0}{u_0}\int_0^{u_0}\left(\frac{\partial Q}{\partial u_0}\right) du = 0 \quad (2)$$

$$Q = -2\pi S\int_r^R r s\,dr = 2\pi S\frac{s_0}{u_0}\frac{a^2}{e^{-2u_0}}\int_u^0 u e^{-2u}du = \frac{-2\pi S a^2 s_0}{u_0 e^{-2u_0}}\left[-\frac{1}{2}ue^{-2u}+\frac{1}{2}\int e^{-2u}du\right]_0^u$$

$$Q = \frac{-\pi S a^2 s_0}{2u_0 e^{-2u_0}}\left(1-e^{-2u}-2ue^{-2u}\right) = \frac{-\pi S a^2 s_0}{2u_0 e^{-2u_0}}\varphi \quad \text{en posant } \varphi = 1-e^{-2u}-2ue^{-2u}$$

$$\frac{\partial Q}{\partial u_0} = \frac{-\pi S a^2 s_0}{2u_0^2 e^{-2u_0}}\left[(2u_0-1)\varphi + u_0\frac{\partial \varphi}{\partial u}\right] \text{ et (2) devient:}$$

$$\dot{u}_0\left(\frac{\pi S a^2 s_0}{2u_0^2 e^{-2u_0}}\right)^2 \underbrace{\int_0^{u_0}\left[(2u_0-1)\varphi+u_0\frac{\partial\varphi}{\partial u}\right]^2 du}_{I_1} + 2\pi T\frac{s_0}{u_0}\left(\frac{-\pi S a^2 s_0}{2u_0^2 e^{-2u_0}}\right)\underbrace{\int_0^{u_0}\left[(2u_0-1)\varphi+u_0\frac{\partial\varphi}{\partial u}\right] du}_{I_4} \quad (3)$$

Calcul de I_1. Une intégration fastidieuse mais facile de :

$$\int_0^{u_0}\left(4u^2e^{-4u}+4u_0^2e^{-4u}+e^{-4u}+4u_0^2+1-8u_0ue^{-4u}+4ue^{-4u}+8u_0ue^{-2u}-4ue^{-2u}-4u_0e^{-4u}-8u_0^2e^{-2u}+4u_0e^{-2u}\right.$$
$$\left.-4u_0e^{-2u}-2e^{-2u}-4u_0\right)du \quad \text{donne :}$$

$$I_1 = 4u_0^3-7u_0^2-4u_0e^{-2u_0}+\frac{11}{2}u_0-\frac{5}{8}e^{-4u_0}+2e^{-2u_0}-\frac{11}{8}$$

Calcul de $I_4 = \int_0^{u_0}\left[(2u_0-1)(1-e^{-2u}-2ue^{-2u})+4u_0\,ue^{-2u}\right]du = 2u_0^2-2u_0-e^{-2u_0}+1$ et (3) devient :

$$\frac{I_1}{4I_4}\,\frac{\dot{u}_0}{u_0} = \frac{T}{Sa^2}e^{-2u_0}. \quad \text{Soit } \alpha = \frac{Tt}{Sa^2} \text{ il vient } \boxed{\frac{du_0}{d\alpha} = \frac{4u_0e^{-2u_0}I_4}{I_1} \text{ et } s = s_0\left(1-\frac{1}{u_0}\log\frac{r}{a}\right)}$$

$$\dot{Q}_0 = \frac{\partial Q_0}{\partial u_0}\dot{u}_0 \quad \text{d'où} \quad \boxed{\frac{\dot{Q}_0}{(-2\pi T)s_0} = \frac{\psi_0 I_4}{u_0 I_1}} \qquad \text{avec } \psi_0 = -1+2u_0+e^{-2u_0}$$

Pour u_0 petit $\dfrac{du_0}{d\alpha} \rightarrow \dfrac{4\cdot\frac{4}{3}u_0^3\cdot u_0}{\frac{32}{15}u_0^5} = \dfrac{5}{2u_0}$ d'où la valeur initiale $u_0 = \sqrt{5\alpha}$

Le problème : $\dfrac{\partial^2 s}{\partial r^2}+\dfrac{1}{r}\dfrac{\partial s}{\partial r} = \dfrac{S}{T}\dfrac{\partial s}{\partial t}$ avec $s(0,t) = s_0$, $s(\infty,t) = 0$, $s(r,0) = 0$

On demande : $s = s(r,t)$ et $\dot{Q}_0 = 2\pi T\left(r\dfrac{\partial s}{\partial r}\right)_{r=a}$

<u>Solution analytique</u>

$\dot{Q}_0 = -2\pi T s_0\,G(\alpha)$ avec :

$$\alpha = \frac{Tt}{Sa^2} \text{ et } G(\alpha) = \frac{4\alpha}{\pi}\int_0^\infty x e^{-\alpha x^2}\left[\frac{\pi}{2}+tg^{-1}\frac{Y_0(x)}{J_0(x)}\right]dx$$

<u>méthode Biot</u>

$\dot{Q}_0 = -2\pi T s_0\dfrac{\psi I_4}{u_0 I_1}$

avec $\dfrac{du_0}{d\alpha} = \dfrac{4u_0 I_4 e^{-2u_0}}{I_1}$

et $\alpha = \dfrac{Tt}{Sa^2}$; $s = \dfrac{s_0}{u_0}\left(u_0-\log\dfrac{r}{a}\right)$

α	u_0	$\frac{\dot{Q}_0}{(-2\pi T)\delta_0}$
10^{-4}	0.02236	59.05079
1.5	0.02724	44.44588
2	0.03129	39.76594
3	0.03813	32.88238
4	0.04388	28.38623
5	0.04892	25.40835
6	0.05346	23.26962
7	0.05762	21.60085
8	0.06147	20.23855
9	0.06509	19.09368
10^{-3}	0.06849	18.14685
1.5	0.08327	14.90848
2	0.09557	12.97644
3	0.11589	10.68343
4	0.13273	9.31597
5	0.14734	8.38244
6	0.16037	7.69313
7	0.17221	7.15729
8	0.18312	6.72526
9	0.19326	6.36731
10^{-2}	0.20276	6.06445
1.5	0.24337	5.03645
2	0.27642	4.42308
3	0.32958	3.69465
4	0.37236	3.25972
5	0.40863	2.96247
6	0.44035	2.74274
7	0.46867	2.57173
8	0.49435	2.43370
9	0.51789	2.31923
10^{-1}	0.53966	2.22226
1.5	0.62987	1.89226
2	0.70012	1.69445
3	0.80797	1.45818
4	0.89067	1.31608
5	0.95825	1.21836
6	1.01565	1.14570
7	1.06565	1.08886
8	1.11004	1.04276
9	1.15000	1.00435
1	1.18636	0.97169
1.5	1.33154	0.85936
2	1.43926	0.79094
3	1.59710	0.70772
4	1.71294	0.65665
5	1.80478	0.62096
6	1.88100	0.59405
7	1.94622	0.57276
8	2.00326	0.55531
9	2.05398	0.54064

α	u_0	$\frac{\dot{Q}_0}{(-2\pi T)\delta_0}$
10	2.09964	0.52806
15	2.27783	0.48396
20	2.40636	0.45635
30	2.59009	0.42182
40	2.72204	0.40002
50	2.82519	0.38445
60	2.90993	0.37252
70	2.98189	0.36295
80	3.04443	0.35501
90	3.09975	0.34827
100	3.14935	0.34244
150	3.34117	0.32158
200	3.47808	0.30816
300	3.67202	0.29093
400	3.81024	0.27978
500	3.91776	0.27167
600	4.00580	0.26536
700	4.08035	0.26025
800	4.14501	0.25597
900	4.20211	0.25230
10^{3}	4.25324	0.24911
1.5	4.45036	0.23751
2	4.59056	0.22989
3	4.78856	0.21992
4	4.92931	0.21334
5	5.03862	0.20849
6	5.12802	0.20469
7	5.20366	0.20158
8	5.26921	0.19896
9	5.32707	0.19670
10^{4}	5.37884	0.19472
1.5	5.57826	0.18746
2	5.71391	0.18262
3	5.91975	0.17621
4	6.06167	0.17191
5	6.17182	0.16873
6	6.26186	0.16620
7	6.33802	0.16413
8	6.40401	0.16237
9	6.46223	0.16086
10^{5}	6.51432	0.15952
1.5	6.71488	0.15458
2	6.85727	0.15126
3	7.05806	0.14680
4	7.20060	0.14380
5	7.31120	0.14155
6	7.40159	0.13976
7	7.47803	0.13829
8	7.54426	0.13703
9	7.60268	0.13594

α	u_0	$\frac{\dot{Q}_0}{(-2\pi T)\delta_0}$
10^{6}	7.65495	0.13499
1.5	7.85616	0.13142
2	7.99897	0.12900
3	8.20032	0.12574
4	8.34322	0.12352
5	8.45408	0.12185
6	8.54468	0.12052
7	8.62129	0.11942
8	8.68766	0.11848
9	8.74621	0.11767
10^{7}	8.79859	0.11695
1.5	9.00019	0.11425
2	9.14326	0.11242
3	9.34495	0.10992
4	9.48809	0.10822
5	9.59912	0.10694
6	9.68986	0.10591
7	9.76658	0.10506
8	9.83304	0.10433
9	9.89167	0.10369
10^{8}	9.94412	0.10313
1.5	10.14598	0.10103
2	10.28923	0.09959
3	10.49115	0.09763
4	10.63444	0.09628
5	10.74559	0.09526
6	10.83641	0.09444
7	10.91321	0.09376
8	10.97974	0.09318
9	11.03842	0.092679
10^{9}	11.09092	0.092231
1.5	11.29296	0.090544
2	11.43633	0.089384
3	11.63841	0.087799
4	11.78181	0.086707
5	11.89304	0.085879
6		
7		
8		
9		
10^{10}	12.23861	0.083404

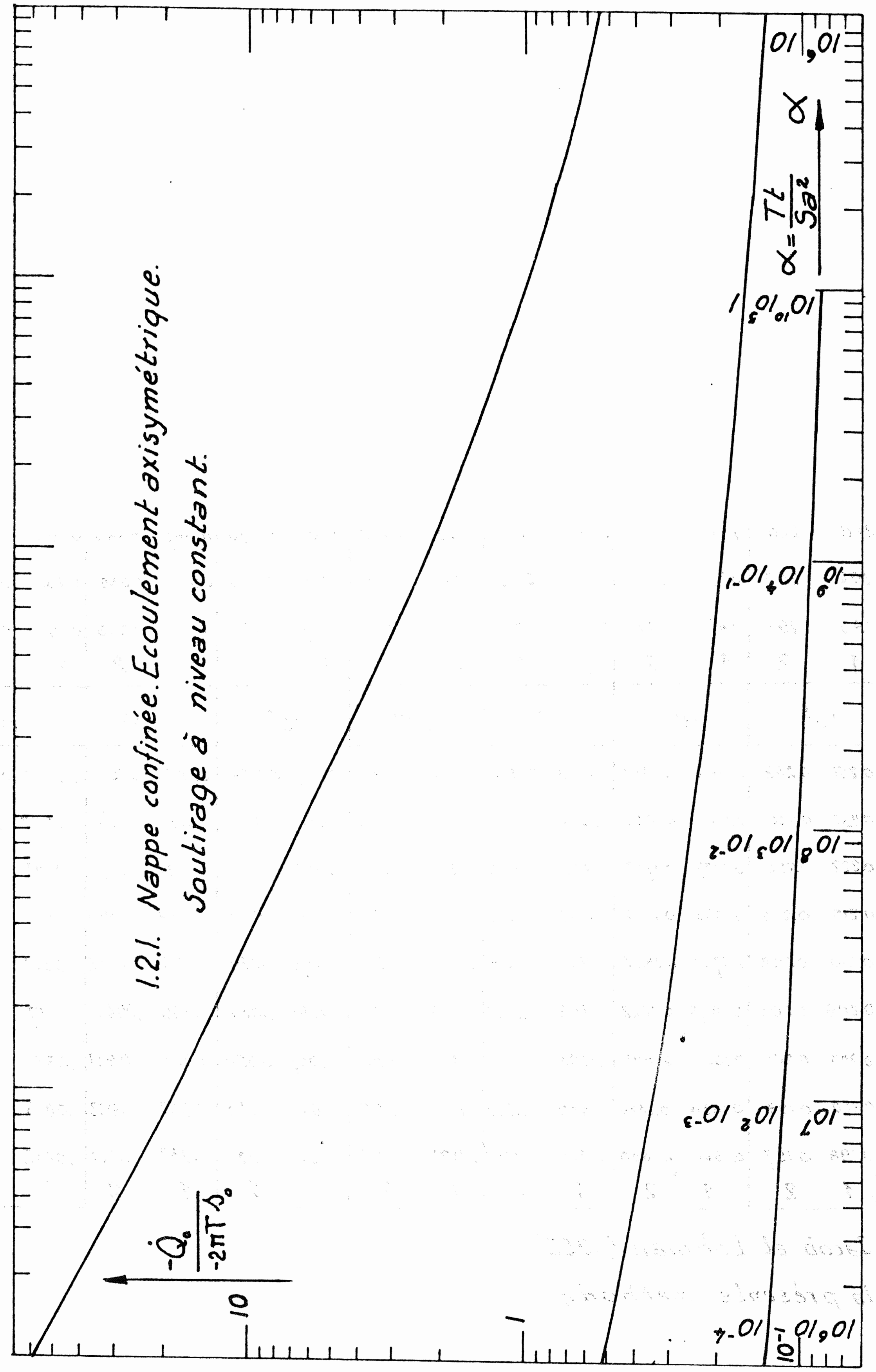
Comparaison des résultats obtenus par la formule analytique de Jacob et Lohman (1952) et de ceux que donne la présente méthode.

	10^{-4}		10^{-3}		10^{-2}		10^{-1}		10^{0}		10^{1}		10^{2}	
1	56.9	59.0	18.34	18.15	6.13	6.06	2.249	2.222	0.985	0.972	0.534	0.528	0.346	0.342
2	40.4	39.7	13.11	12.98	4.47	4.42	1.716	1.694	0.803	0.791	0.461	0.456	0.311	0.308
3	33.1	32.9	10.79	10.68	3.74	3.69	1.477	1.458	0.719	0.708	0.427	0.422	0.294	0.291
4	28.7	28.4	9.41	9.31	3.30	3.26	1.333	1.316	0.667	0.657	0.405	0.400	0.283	0.280
5	25.7	25.4	8.47	8.38	3.00	2.96	1.234	1.218	0.630	0.621	0.389	0.384	0.274	0.272
6	23.5	23.3	7.77	7.69	2.78	2.74	1.160	1.146	0.602	0.594	0.377	0.372	0.268	0.265
7	21.8	21.6	7.23	7.16	2.60	2.57	1.103	1.089	0.580	0.573	0.367	0.363	0.263	0.260
8	20.4	20.2	6.79	6.72	2.46	2.43	1.057	1.043	0.562	0.555	0.359	0.355	0.258	0.255
9	19.3	19.1	6.43	6.37	2.35	2.32	1.018	1.004	0.547	0.541	0.352	0.348	0.254	0.252
	1	2	1	2	1	2	1	2	1	2	1	2	1	2

	10^{3}		10^{4}		10^{5}		10^{6}		10^{7}		10^{8}		10^{9}	
1	0.251	0.249	0.1964	0.1947	0.1608	0.1595	0.1360	0.1350	0.1177	0.1169	0.1037	0.1031	0.0927	0.0922
2	0.232	0.230	0.1841	0.1826	.1524	.1512	.1299	.1290	.1131	.1124	.1002	.0996	.0899	.0894
3	0.222	0.220	0.1777	0.1762	.1479	.1468	.1266	.1257	.1106	.1099	.0982	.0976	.0883	.0878
4	0.215	0.213	0.1733	0.1719	.1449	.1438	.1244	.1235	.1089	.1082	.0968	.0963	.0872	.0867
5	0.210	0.208	0.1701	0.1687	.1426	.1415	.1227	.1218	.1076	.1069	.0958	.0953	.0864	.0859
6	0.206	0.205	0.1675	0.1662	.1408	.1398	.1213	.1205	.1066	.1059	.0950	.0944	.0857	
7	0.203	0.201	0.1654	0.1641	.1393	.1383	.1202	.1194	.1057	.1050	.0943	.0937	.0851	
8	0.200	0.199	0.1636	0.1624	.1380	.1370	.1192	.1185	.1049	.1043	.0937	.0932	.0846	
9	0.198	0.197	0.1621	0.1608	.1369	.1359	.1184	.1177	.1043	.1037	.0932	.0927	.0842	
	1	2	1	2	1	2	1	2	1	2	1	2	1	2

1 Jacob et Lohman (1952)

2 la présente méthode.



1.2.1. Nappe confinée. Ecoulement axisymétrique.
Soutirage à niveau constant.

1.2.2. Nappe confinée. Ecoulement axisymétrique. Soutirage à débit constant : $-\dot{Q}_0$.

Solution analytique. (Theis, 1935)

$$s = \frac{-\dot{Q}_0}{4\pi T}\int_u^{\infty}\frac{e^{-u}}{u}\,du \qquad \text{avec } u = \frac{Sr^2}{4Tt} \qquad \text{ou} \quad s = -\frac{-\dot{Q}_0}{4\pi T}E_1(u).$$

Méthode pseudo variationnelle.

Continuité : $\frac{d\dot{Q}}{dr} = 2\pi r S\dot{s}$ ou $\frac{dQ}{dr} = 2\pi r S s$

Darcy : $\dot{Q} = 2\pi T r\frac{ds}{dr}$ ou $\int_a^R \dot{Q}\,\delta Q\,\frac{dr}{r} = 2\pi T\int_a^R \frac{ds}{dr}\,\delta Q\,dr$ (') avec $\begin{cases} \dot{Q} = \frac{\partial Q}{\partial R}\dot{R} + \frac{\partial Q}{\partial s_0}\dot{s}_0 \\ \delta Q = \frac{\partial Q}{\partial R}\delta R + \frac{\partial Q}{\partial s_0}\delta s_0 \end{cases}$

Soit $s = \frac{s_0}{u_0}u$ avec $\begin{cases} u = \log\frac{R}{r} \\ u_0 = \log\frac{R}{a} \end{cases}$ d'où $\begin{cases} r = Re^{-u} = a\frac{e^{-u}}{e^{-u_0}} \; ; \; dr = -a\frac{e^{-u}}{e^{-u_0}}du \\ r\,dr = -a^2\frac{e^{-2u}}{e^{-2u_0}}du \; ; \; \frac{ds}{dr} = -a\frac{s_0}{u_0}\frac{e^{-u}}{e^{-u_0}} \; ; \; \frac{dr}{r} = -du. \end{cases}$

$$Q = -2\pi S\int_r^R rs\,dr = \frac{-\pi S a^2 s_0 u_0}{2u_0^2 e^{-2u_0}}\varphi \quad \text{(calculé antérieurement)}$$

$$\frac{\partial Q}{\partial u_0} = \frac{-\pi S a^2 s_0}{2u_0^2 e^{-2u_0}}\left[u_0\frac{\partial\varphi}{\partial u} + (2u_0-1)\varphi\right] \quad \text{et} \quad \frac{\partial Q}{\partial s_0} = \frac{-\pi S a^2 u_0}{2u_0^2 e^{-2u_0}}\varphi.$$

$$\left.\begin{aligned} \frac{\partial Q_0}{\partial u_0} &= \frac{-\pi S a^2 s_0}{2u_0^2 e^{-2u_0}}\psi_0 \\ \frac{\partial Q_0}{\partial s_0} &= \frac{-\pi S a^2 u_0}{2u_0^2 e^{-2u_0}}\varphi_0 \end{aligned}\right\} \quad \dot{Q}_0 = \frac{-\pi S a^2}{2u_0^2 e^{-2u_0}}u_0 s_0\left(\psi_0\frac{\dot{u}_0}{u_0} + \varphi\frac{\dot{s}_0}{s_0}\right)$$

$$\dot{Q} = \frac{-\pi S a^2}{2u_0^2 e^{-2u_0}}\left[s_0\dot{u}_0\left[u_0\frac{\partial\varphi}{\partial u} + (2u_0-1)\varphi\right] + u_0\dot{s}_0\varphi\right]$$ et (') devient :

$$\int_a^R \dot{Q}\frac{\partial Q}{\partial R}\frac{dr}{r} = 2\pi T\int_a^R \frac{ds}{dr}\frac{\partial Q}{\partial R}\,dr \quad \text{ou} \quad \int_0^{u_0}\dot{Q}\frac{\partial Q}{\partial u_0}du = -2\pi T\frac{s_0}{u_0}\int_0^{u_0}\frac{\partial Q}{\partial u_0}du \quad (^2)$$

$$\dot{Q}\frac{\partial Q}{\partial u_0} = \left(\frac{\pi S a^2}{2u_0^2 e^{-2u_0}}\right)^2\left\{s_0^2\dot{u}_0\left[u_0\frac{\partial\varphi}{\partial u} + (2u_0-1)\varphi\right]^2 + s_0 u_0\dot{s}_0\varphi\left[u_0\frac{\partial\varphi}{\partial u} + (2u_0-1)\varphi\right]\right\}$$

et (²) devient :

$$\frac{I_1}{I_4}\frac{\dot u_0}{u_0}+\frac{I_2}{I_4}\frac{\dot s_0}{s_0}=\frac{-4Te^{-2u_0}}{Sa^2} \quad \text{avec :} \qquad (^3)$$

$$\left.\begin{array}{l} I_1=\displaystyle\int_0^{u_0}\left[u_0\frac{\partial\varphi}{\partial u}+(2u_0-1)\varphi\right]^2du=4u_0^3-7u_0^2-4u_0e^{-2u_0}+\frac{11}{2}u_0-\frac{5}{8}e^{-4u_0}+2e^{-2u_0}-\frac{11}{8} \\ I_4=\displaystyle\int_0^{u_0}\left[u_0\frac{\partial\varphi}{\partial u}+(2u_0-1)\varphi\right]du=2u_0^2-2u_0-e^{-2u_0}+1. \end{array}\right\}\text{calculés antérieurement}$$

$$I_2=\int_0^{u_0}\varphi\left[u_0\frac{\partial\varphi}{\partial u}+(2u_0-1)\varphi\right]du=\int_0^{u_0}\left(-1+2e^{-2u}+4ue^{-2u}+2u_0-4u_0e^{-2u}-e^{-4u}-4ue^{-4u}+2u_0e^{-4u}-4u^2e^{-4u}\right.$$
$$\left.-4u_0ue^{-2u}+4u_0ue^{-4u}\right)du=2u_0^2e^{-2u_0}+2u_0^2+\frac{3}{4}u_0e^{-4u_0}+u_0e^{-2u_0}-\frac{13}{4}u_0+\frac{5}{8}e^{-4u_0}-2e^{-2u_0}+\frac{11}{8}$$

La contrainte $-\dot Q_0=$ Constante s'écrit $\dot Q_0 t=Q_0$ ou :

$$\frac{-\pi Sa^2}{2u_0^2e^{-2u_0}}u_0s_0\left(\psi_0\frac{\dot u_0}{u_0}+\varphi_0\frac{\dot s_0}{s_0}\right)t=\frac{-\pi Sa^2 s_0u_0}{2u_0^2e^{-2u_0}}\varphi_0 \quad \text{ou}$$

$$\frac{\dot s_0}{s_0}=\frac{1}{t}-\frac{\psi_0}{\varphi_0}\frac{\dot u_0}{u_0} \quad \text{et } (^3) \text{ devient :}$$

$$\frac{\dot u_0}{u_0}\left(\frac{I_1}{I_4}-\frac{I_2}{I_4}\frac{\psi_0}{\varphi_0}\right)+\frac{I_2}{I_4}\frac{1}{t}=\frac{4T}{Sa^2}e^{-2u_0}. \quad \text{Soit } \boxed{\frac{4Tt}{Sa^2}=\alpha}\text{, il vient :}$$

$$\frac{1}{u_0}\frac{I_1\varphi_0-I_2\psi_0}{I_4\varphi_0}\frac{du_0}{d\alpha}+\frac{I_2}{I_4}\frac{1}{\alpha}=e^{-2u_0} \quad \text{Soit } F=\frac{I_1\varphi_0-I_2\psi_0}{u_0}$$

$$F=-8u_0^3e^{-2u_0}+6u_0^2e^{-2u_0}+\frac{9}{2}u_0e^{-4u_0}-6u_0e^{-2u_0}+\frac{3}{2}u_0+\frac{1}{2}e^{-6u_0}-\frac{3}{2}e^{-4u_0}+\frac{3}{2}e^{-2u_0}-\frac{1}{2}$$

d'où $\boxed{\dfrac{du_0}{d\alpha}=\dfrac{(I_4e^{-2u_0}\alpha-I_2)}{F\alpha}\varphi_0}$ $(^4)$ $\dot Q_0 t=\dfrac{-\pi Sa^2 s_0\varphi_0}{2u_0e^{-2u_0}}$ donne :

$$\boxed{\frac{s_0}{\left(\frac{-\dot Q_0}{4\pi T}\right)}=\frac{2u_0e^{-2u_0}}{\varphi_0}\alpha=W(\alpha)}$$

Pour u_0 petit $\dfrac{du_0}{d\alpha}=\dfrac{\left(\frac{4}{3}u_0^3\alpha-\frac{6}{5}u_0^5\right)2u_0^2}{\frac{28}{15}u_0^6\alpha}=\dfrac{10}{7u_0}-\dfrac{9}{7}\dfrac{u_0}{\alpha}$ dont la solution est : $u_0=\sqrt{\frac{4}{5}\alpha}$

$s=s_0\left(1-\frac{1}{u_0}\log\frac{r}{\alpha}\right)$

La formule analytique à laquelle cette solution devrait être comparée n'est pas celle de Theis mais la formule (17) de la page 338 de Carslaw and Jaeger :

$$s=\frac{-\dot Q_0}{4\pi T}S(\tau,\rho) \text{ avec } S(\tau,\rho)=\frac{4}{\pi}\int_0^{\infty}(1-e^{-\tau u^2})\frac{J_1(u)Y_0(\rho u)-Y_1(u)J_0(\rho u)}{u^2[J_1^2(u)+Y_1^2(u)]}du$$

$\tau=\dfrac{Tt}{Sa^2}$ et $\rho=\dfrac{r}{a}$

Si $\rho=1$ $S(\tau,\rho)=\dfrac{8}{\pi^2}\displaystyle\int_0^{\infty}\frac{1-e^{-\tau u^2}}{u^3[J_1^2(u)+Y_1^2(u)]}du$ car $J_1Y_0-J_0Y_1=\dfrac{2}{\pi u}$

Formule de Evertingen and Hurst (1949)

Dans $(^4)$, posons $\zeta=e^{2u_0}$, il vient $\dfrac{d\zeta}{d\alpha}=\dfrac{2I_4\varphi}{F}-\dfrac{2I_2\varphi}{F}\dfrac{\zeta}{\alpha}$. Pour ζ grand, il faut trouver $\dfrac{\alpha}{\zeta}=\dfrac{\alpha}{e^{2u_0}}=1-\dfrac{\gamma'}{\log\alpha}$ où γ' est proche de la constante d'Euler.

d'où $\dfrac{s_0}{\left(\frac{-\dot Q_0}{4\pi T}\right)}=\log\alpha-\gamma'$. Pour $\alpha=10^6$, on trouve $\gamma'=0.5573586$.

1.

α	u_0	$\dfrac{\Delta_0}{-Q_0/4\pi T}$
10^{-3}	0.02828	0.03469
1.5	0.03475	0.04217
2	0.03964	0.04913
3	0.04808	0.06040
4	0.05524	0.06976
5	0.06158	0.07789
6	0.06728	0.08522
7	0.07251	0.09193
8	0.07735	0.09815
9	0.08188	0.10399
10^{-2}	0.08615	0.10950
1.5	0.10466	0.13349
2	0.12004	0.15354
3	0.14537	0.18685
4	0.16631	0.21460
5	0.18443	0.23881
6	0.20058	0.26051
7	0.21522	0.28030
8	0.22868	0.29858
9	0.24118	0.31564
10^{-1}	0.25288	0.33168
1.5	0.30272	0.40074
2	0.34308	0.45757
3	0.40766	0.55016
4	0.45934	0.62569
5	0.50294	0.69043
6	0.54093	0.74757
7	0.57473	0.79900
8	0.60528	0.84594
9	0.63322	0.88924
1	0.65900	0.92951
1.5	0.76518	1.09860
2	0.84723	1.23266
3	0.97215	1.44219
4	1.06715	1.60570
5	1.14431	1.74099
6	1.20952	1.85698
7	1.26611	1.95879
8	1.31617	2.04970
9	1.36110	2.13195
10	1.40189	2.20713

α	u_0	$\dfrac{\Delta_0}{-Q_0/4\pi T}$
10	1.40189	2.20713
1.5	1.56378	2.50998
2	1.68299	2.73701
3	1.85637	3.07234
4	1.98272	3.31986
5	2.08240	3.51668
6	2.16481	3.68031
7	2.23511	3.82041
8	2.29644	3.94298
9	2.35084	4.05196
10^{2}	2.39974	4.15008
1.5	2.58971	4.53269
2	2.72602	4.80815
3	2.91987	5.20087
4	3.05844	5.48193
5	3.16641	5.70106
6	3.25489	5.88085
7	3.32988	6.03309
8	3.39496	6.16520
9	3.45246	6.28189
10^{3}	3.50394	6.38640
1.5	3.70256	6.78961
2	3.84389	7.07620
3	4.04353	7.48105
4	4.18546	7.76843
5	4.29567	7.99153
6	4.38577	8.17437
7	4.46202	8.32850
8	4.52811	8.46203
9	4.58643	8.57985
10^{4}	4.63861	8.68529
1.5	4.83957	9.09118
2	4.98225	9.37946
3	5.18349	9.78594
4	5.32639	10.07380
5	5.43727	10.29723
6	5.52783	10.48112
7	5.60448	10.63505
8	5.67090	10.76848
9	5.72948	10.88627
10^{5}	5.78189	10.99169

α	u_0	$\dfrac{\Delta_0}{-Q_0/4\pi T}$
10^{5}	5.78189	10.99169
1.5	5.98358	11.39843
2	6.12678	11.68600
3	6.32859	12.09284
4	6.47188	12.38024
5	6.58303	12.60352
6	6.67372	12.78397
7	6.75057	12.94208
8	6.81713	13.07503
9	6.87583	13.19258
10^{6}	6.92834	13.29786
1.5	7.13036	13.70503
2	7.27380	13.99193
3	7.47589	14.39931
4	7.61938	14.68593
5	7.73066	14.90898
6	7.82138	15.09739
7	7.89835	15.24891
8	7.96501	15.38106
9	8.02378	15.49822
10^{7}	8.07635	15.60328
5	8.87935	17.21379
10^{8}	9.22525	17.90823
5	10.02868	19.51819
10^{9}	10.37473	20.21297
5	11.17845	21.82233
10^{10}	11.52459	22.51769
5	12.32852	24.12632
10^{11}	12.67471	24.82266
5	13.47881	26.43027
10^{12}	13.82503	27.12838
10^{13}	14.97544	29.43765
10^{14}	16.12548	31.77767
10^{15}	17.27303	34.29489
10^{16}	18.41716	37.09440
10^{17}	19.71236	
10^{18}	20.94707	

α	R/a
10^{-1}	1.2877
1	1.9328
10	4.0629
10^{2}	11.0203
10^{3}	33.2464

α	R/a
10^{4}	103.4013
10^{5}	324.3721
10^{6}	1020.8031
10^{7}	3217.4732
10^{8}	10150.2756

α	R/a
10^{9}	32039.69
10^{10}	101173.26
10^{11}	319565.56
10^{12}	1009569.43
10^{16}	99648814.71

1.2.2. Comparaison entre les résultats obtenus par l'emploi de la formule Everdingen and Hurst (1949) et ceux que donne la présente méthode.

$\alpha:4$	$s_0/(-\dot{Q}_0/4\pi T)$ 1	2	α	$s_0/(-\dot{Q}_0/4\pi T)$ 1	2
0.10	0.616	0.625	6	2.865	2.886
0.20	0.842	0.845	8	3.104	3.127
0.30	1.005	1.003	10	3.305	3.319
0.40	1.131	1.128	12	3.456	3.480
0.50	1.244	1.232	15	3.657	3.680
0.60	1.344	1.324	20	3.921	3.943
0.70	1.420	1.405	25	4.122	4.150
0.80	1.483	1.478	30	4.298	4.321
1.00	1.608	1.606	50	4.775	4.808
1.20	1.722	1.716	100	5.441	5.482
1.50	1.860	1.857	500	7.037	7.076
2	2.048	2.050	1000	7.716	7.768
2.5	2.199	2.207	5000	9.324	9.379
3.	2.337	2.341	10000	10.015	10.074
4.	2.551	2.560	25000	10.933	10.991
5	2.727	2.737			

1 Formule Everdingen et Hurst (1949) tabulée dans Hantush (1964), p 319.
2 la présente méthode.

$$\frac{s}{-\frac{Q_o}{4\pi T}} = \frac{2u_o e^{-2u_o}\alpha}{\varphi_o}\left(1-\frac{\log \rho}{u_o}\right) = \frac{s_o}{-\frac{Q_o}{4\pi T}}\left(1-\frac{\log \rho}{u_o}\right) \qquad \rho = \frac{r}{a} \qquad \alpha = \frac{4Tt}{Sa^2}$$

α	$s/(-Q_o/4\pi T)$			
	$\rho = 1$	$\rho = 2$	$\rho = 5$	$\rho = 10$
1.5	1.0986	0.1034		
2	1.2326	0.2242		
5	1.7410	0.6864		
10	2.2071	1.1158		
20	2.7370	1.6097	0.11962	
40	3.3198	2.1592	0.62502	
60	3.6803	2.5019	0.94416	
80	3.9430	2.7528	1.17958	
100	4.1501	2.9513	1.36674	0.1680
200	4.8081	3.5855	1.9694	0.7468
400	5.4819	4.2395	2.5972	1.3548
2000	7.0762	5.8001	4.1134	2.8374
4000	7.7684	6.4819	4.7812	3.4947
20000	9.3794	8.0745	6.3496	5.0446
40000	10.0738	8.7628	7.0299	5.7189
10^5	10.9917	9.67398	7.9321	6.6143

à comparer au tableau incomplet publié par Hantush (1964) p319. table VI : fonction : $S(\tau, \rho)$

Le problème : $\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} = \frac{S}{T}\frac{\partial s}{\partial t}$ avec $s(\infty,t)=0$; $s(r,0)=0$; $2\pi T\left(r\frac{\partial s}{\partial r}\right)_{r=a} = \dot{Q}_o$

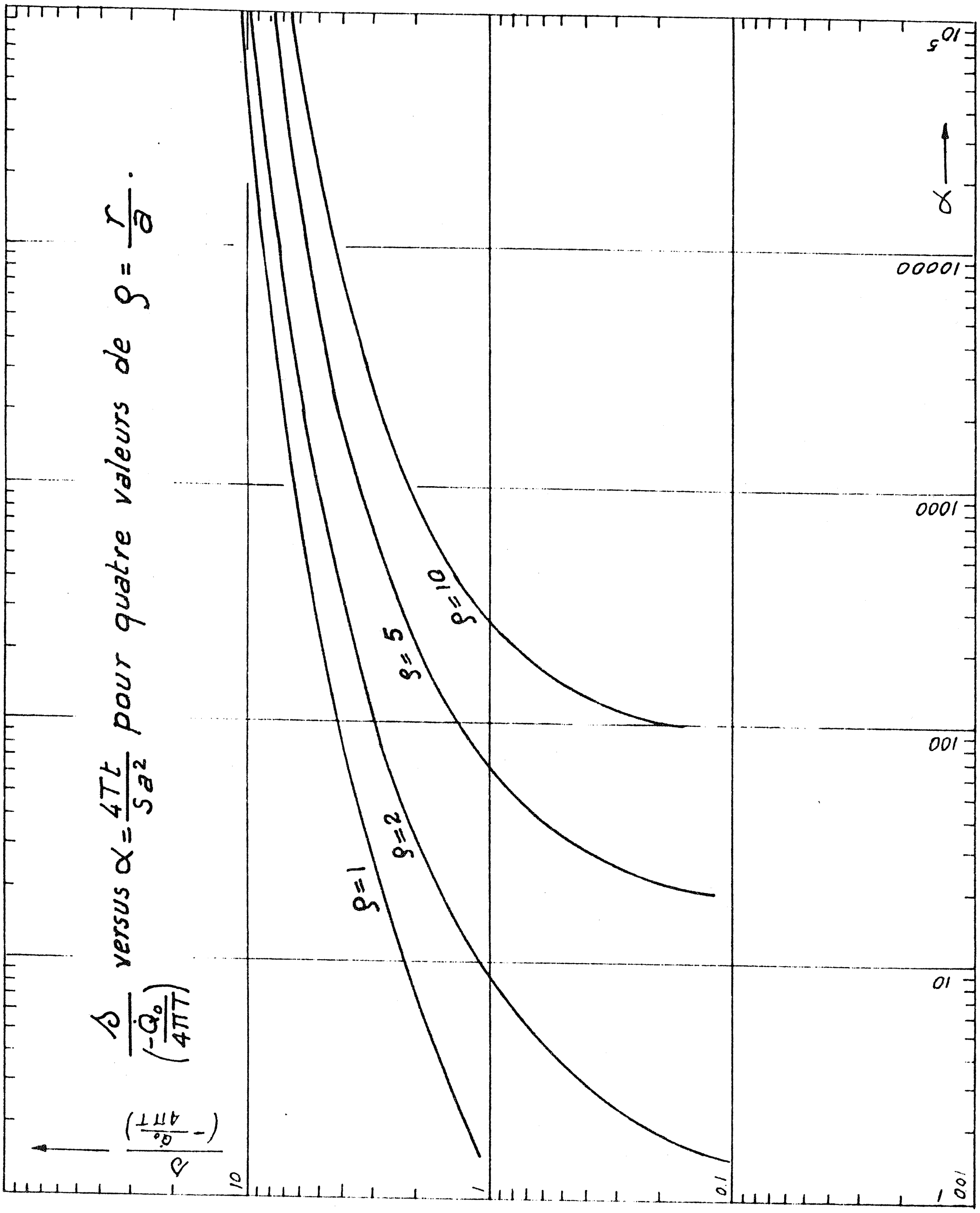
Solution : $\frac{du_o}{d\alpha} = \frac{(I_1 e^{-2u_o}\alpha - I_2)\,\varphi}{F\alpha}$; $\frac{s_o}{\left(-\frac{Q_o}{4\pi T}\right)} = \frac{2u_o e^{-2u_o}}{\varphi}\alpha$

et $s = s_o\left(1-\frac{1}{u_o}\log\frac{r}{a}\right)$

Solution analytique Theis (1935) $s = \frac{-\dot{Q}_o}{4\pi T}\int_u^\infty \frac{e^{-u}}{u}du$ avec $u = \frac{Sr^2}{4Tt}$ ou, mieux.

Carslaw and Jaeger (1959) $s = \frac{-\dot{Q}_o}{4\pi T}\,\frac{4}{\pi}\int_0^\infty (1-e^{-\tau u^2})\frac{J_1(u)Y_0(\rho u)-Y_1(u)J_0(\rho u)}{u^2[J_1^2(u)+Y_1^2(u)]}du$

$\tau = \frac{Tt}{Sa^2}$ et $\rho = \frac{r}{a}$


$\frac{s}{\left(\frac{-Q_0}{4\pi T}\right)}$ versus $\alpha = \frac{4Tt}{Sa^2}$ pour quatre valeurs de $\rho = \frac{r}{a}$.
$\rho=1$
$\rho=2$
$\rho=5$
$\rho=10$
$\frac{s}{\left(-\frac{Q_0}{4\pi T}\right)}$
α
10
1
0.1
100
10
1000
10000
10^5

2.1.1. Nappe libre. Ecoulement parallèle. Soutirage à niveau constant s_0.
(Hypothèse Dupuit-Forchheimer)

Continuité : $\frac{d\dot{Q}}{dx} = -f\dot{h}$ ou $\frac{dQ}{dx} = fs$

Darcy : $\dot{Q} + kh\frac{dh}{dx} = 0$ ou $\dot{Q} = k(h_1 - s)\frac{ds}{dx}$ ou

$\int_0^q \dot{Q}\,\delta Q\,dx = k\int_0^q (h_1 - s)\frac{ds}{dx}\,\delta Q\,dx$ avec $\dot{Q} = \frac{\partial Q}{\partial q}\dot{q}$ et $\delta Q = \frac{\partial Q}{\partial q}\delta q$ d'où

$$\dot{q}\int_0^q \left(\frac{\partial Q}{\partial q}\right)^2 dx = k\int_0^q (h_1 - s)\frac{ds}{dx}\frac{\partial Q}{\partial q}\,dx \qquad (')$$

Soit $s = s_0\zeta^2$ avec $\zeta = 1 - \frac{x}{q}$. On a : $\begin{cases} \frac{d\zeta}{dx} = -\frac{1}{q} & ;\ dx = -q\,d\zeta \\ \frac{ds}{dx} = 2s_0\zeta\frac{d\zeta}{dx} = -2s_0\frac{\zeta}{q} & ;\ \frac{d\zeta}{dq} = \frac{x}{q^2} = \frac{1-\zeta}{q} \end{cases}$

$$(') \text{ devient : } q\dot{q} \times \int_0^1 \left(\frac{\partial Q}{\partial q}\right)^2 d\zeta = -2ks_0\int_0^1 (h_1 - s_0\zeta^2)\zeta\frac{\partial Q}{\partial q}\,d\zeta \qquad (^2)$$

$$\text{Or } Q = -f\int_x^q s\,dx = fq\int_\zeta^0 s\,d\zeta = -fqs_0\int_0^\zeta \zeta^2 d\zeta = -\frac{1}{3}fqs_0\zeta^3 \quad \therefore Q_0 = -\frac{1}{3}fqs_0$$

$$\frac{\partial Q}{\partial q} = -\frac{1}{3}fs_0\left(\zeta^3 + 3q\zeta^2\frac{\partial\zeta}{\partial q}\right) = -\frac{1}{3}fs_0(3\zeta^2 - 2\zeta^3)$$

$$\int_0^1 \left(\frac{\partial Q}{\partial q}\right)^2 d\zeta = \frac{1}{9}f^2s_0^2\int_0^1 (3\zeta^2 - 2\zeta^3)^2 d\zeta = \frac{1}{9}f^2s_0^2\left[\frac{9}{5} - \frac{12}{6} + \frac{4}{7}\right] = \frac{13}{9\times 35}f^2s_0^2$$

$$\int_0^1 (h_1 - s_0\zeta^2)\zeta\frac{\partial Q}{\partial q}\,d\zeta = -\frac{1}{3}fs_0\int_0^1 (h_1 - s_0\zeta^2)\zeta(3\zeta^2 - 2\zeta^3)\,d\zeta = -\frac{1}{3}fs_0\int_0^1 (3h_1\zeta^3 - 2h_1\zeta^4 - 3s_0\zeta^5 + 2s_0\zeta^6)\,d\zeta$$

$= -\frac{1}{3}fs_0\left(\frac{7}{20}h_1 - \frac{3}{14}s_0\right)$ et $(^2)$ devient : $\frac{13}{9.35}f^2s_0^2 q\dot{q} = ks_0^2 f\frac{2}{13}\left(\frac{7}{20}h_1 - \frac{3}{14}s_0\right)$ ou

$q\dot{q} = \frac{3}{26}\frac{k}{f}(49h_1 - 30s_0)$ et $q^2 = \frac{3}{13}\frac{kt}{f}(49h_1 - 30s_0)$

$s = s_0\zeta^2 = s_0\left(1 - \frac{x}{\sqrt{\frac{3}{13}\frac{kh_1t}{f}\left(49 - 30\frac{s_0}{h_1}\right)}}\right)^2$ ou, en posant $u^2 = \frac{fx^2}{4kh_1t}$

$$\boxed{s = s_0\left(1 - \frac{u}{\sqrt{\frac{147}{52}\left(1 - \frac{30}{49}\frac{s_0}{h_1}\right)}}\right)^2 = s_0 D(u)}$$

et $\dot{Q}_o = \frac{\partial Q}{\partial q}\,\dot{q} = -\frac{1}{3} f s_o \sqrt{\frac{kh_1}{ft}} \sqrt{\frac{3}{52}\left(49 - 30\frac{s_o}{h_1}\right)} = -s_o \sqrt{1 - \frac{30}{49}\frac{s_o}{h_1}} \sqrt{\frac{kh_1 f}{\frac{159}{49}(=3.1837)\,t}}$

Dans le cas d'une nappe libre, $\dot{Q}_o$ n'est plus fonction linéaire de s_o.

En posant $\alpha = \frac{kh_1 t}{f}$ et $\sigma_o = \frac{s_o}{h_1}$

$$\boxed{\frac{\dot{Q}_o}{-\sigma_o k h_1^2 \sqrt{1 - \frac{30}{49}\sigma_o}} = \frac{1}{\sqrt{\approx \pi \alpha}}}$$

u	$D(u)$					
	$\sigma_o = 0.01$	$\sigma_o = 0.1$	$\sigma_o = \frac{1}{3}$	$\sigma_o = \frac{1}{2}$	$\sigma_o = \frac{2}{3}$	$\sigma_o = 1$
0.01	0.9880	0.9873	0.9851	0.9829	0.9800	0.9695
0.02	0.9762	0.9748	0.9703	0.9660	0.9602	0.9396
0.05	0.9410	0.9376	0.9266	0.9161	0.9020	0.8525
0.1	0.8839	0.8773	0.8561	0.8359	0.8091	0.7167
0.2	0.7749	0.7626	0.7234	0.6865	0.6384	0.4805
0.5	0.4910	0.4668	0.3923	0.3265	0.2475	0.0543
1	0.1612	0.1343	0.0639	0.0204		
1.2	0.0794	0.0575	0.0106			
1.5	0.0105	0.0024				

Le problème : $(h_1 - s)\frac{\partial^2 s}{\partial x^2} - \left(\frac{\partial s}{\partial x}\right)^2 = \frac{f}{k}\frac{\partial s}{\partial t}$ avec $s(0,t) = s_o$
$s(x,0) = 0$
$s(\infty,t) = 0$

On demande $s = s(x,t)$?

$$\dot{Q}_o = k(h_1 - s_o)\left(\frac{\partial s}{\partial x}\right)_{x=0}$$

Solution: $s = s_o\,D(u) = s_o\left(1 - \sqrt{\frac{52}{147}}\,\frac{u}{\sqrt{1 - \frac{30}{49}\frac{s_o}{h_1}}}\right)^2$ avec $u^2 = \frac{fx^2}{4kh_1 t}$

$$\dot{Q}_o = -s_o\sqrt{1 - \frac{30}{49}\frac{s_o}{h_1}} \cdot \sqrt{\frac{kh_1 f}{\frac{156}{49}\,t}}$$

2.1.2. Nappe libre. Ecoulement parallèle. Soutirage à débit constant : $-\dot{Q}_o$ (Débit en provenance d'un seul côté)

Continuité : $\frac{d\dot{Q}}{dx} = -f\dot{h}$ ou $\frac{dQ}{dx} = fs$

Darcy : $\dot{Q} + kh\frac{dh}{dx} = 0$ ou $\dot{Q} = k(h_1 - s)\frac{ds}{dx}$ ou $\int_0^q \dot{Q}\,\delta Q\,dx = k\int_0^q (h_1 - s)\frac{ds}{dx}\,\delta Q\,dx$ avec :

$\dot{Q} = \frac{\partial Q}{\partial q}\dot{q} + \frac{\partial Q}{\partial s_o}\dot{s}_o$ et $\delta Q = \frac{\partial Q}{\partial q}\delta q + \frac{\partial Q}{\partial s_o}\delta s_o$ d'où :

$$\int_0^q \dot{Q}\frac{\partial Q}{\partial q}\,dx = k\int_0^q (h_1 - s)\frac{ds}{dx}\frac{\partial Q}{\partial q}\,dx \quad (')$$

Soit $s = s_o\zeta^2$ avec $\zeta = 1 - \frac{x}{q}$. On a : $\left\{\begin{array}{l} \frac{ds}{dx} = 2s_o\zeta\frac{d\zeta}{dx} \; ; \; \frac{d\zeta}{dx} = -\frac{1}{q} \; ; \; dx = -q\,d\zeta \\ \frac{d\zeta}{dq} = \frac{x}{q^2} = \frac{1-\zeta}{q} \end{array}\right.$ et (') devient :

$$q\int_0^1 \dot{Q}\frac{\partial Q}{\partial q}\,d\zeta = -k\int_0^1 (h_1 - s)\frac{\partial Q}{\partial q}\frac{ds}{d\zeta}\,d\zeta \quad (^2)$$

$$Q = -f\int_x^q s\,dx = fs_oq\int_\zeta^0 \zeta^2 d\zeta = -\tfrac{1}{3}fs_oq\zeta^3 \quad \therefore \quad Q_o = -\tfrac{1}{3}fs_oq$$

$\frac{\partial Q}{\partial q} = -\frac{1}{3}fs_o\left[\zeta^3 + 3q\zeta^2\frac{\partial\zeta}{\partial q}\right] = -\frac{1}{3}fs_o(3\zeta^2 - 2\zeta^3)$ et $\frac{\partial Q}{\partial s_o} = -\frac{1}{3}fq\zeta^3$ d'où

$$\dot{Q} = -\tfrac{1}{3}f\zeta^2\left[s_o(3-2\zeta)\dot{q} + \zeta q\dot{s}_o\right] = -\tfrac{1}{3}fs_oq\zeta^2\left[(3-2\zeta)\frac{\dot{q}}{q} + \zeta\frac{\dot{s}_o}{s_o}\right]$$

α) calcul de $\int_0^1 \dot{Q}\frac{\partial Q}{\partial q}\,d\zeta = (-\frac{1}{3}fs_o)^2 q\int_0^1 \zeta^4\left[(3-2\zeta)\frac{\dot{q}}{q} + \zeta\frac{\dot{s}_o}{s_o}\right](3-2\zeta)\,d\zeta$

$$= (-\tfrac{1}{3}fs_o)^2 q\left[\frac{\dot{q}}{q}\int_0^1 \zeta^4(3-2\zeta)^2 d\zeta + \frac{\dot{s}_o}{s_o}\int_0^1 \zeta^5(3-2\zeta)\,d\zeta\right]$$

$$= (-\tfrac{1}{3}fs_o)^2 q\left[\frac{13}{35}\frac{\dot{q}}{q} + \frac{3}{14}\frac{\dot{s}_o}{s_o}\right]$$

β) calcul de $\int_0^1 (h_1 - s)\frac{\partial Q}{\partial q}\frac{ds}{d\zeta}\,d\zeta = -\frac{2}{3}fs_o^2\int_0^1 (3\zeta^2 - 2\zeta^3)(h_1 - s_o\zeta^2)\zeta\,d\zeta$

$= -\frac{2}{3}fs_o^2\left[\frac{3}{4}h_1 - \frac{3}{6}s_o - \frac{2}{5}h_1 + \frac{2}{7}s_o\right] = -\frac{1}{3}fs_o^2\left[\frac{7}{10}h_1 - \frac{3}{7}s_o\right]$ et (²) devient :

$$q^2(\tfrac{1}{3}fs_o)^2\left[\frac{13}{35}\frac{\dot{q}}{q} + \frac{3}{14}\frac{\dot{s}_o}{s_o}\right] = \tfrac{1}{3}kfs_o^2\left(\frac{7}{10}h_1 - \frac{3}{7}s_o\right) \quad (^3)$$

La contrainte $-\dot{Q}_o$ = Constante s'écrit : $\dot{Q}_o t = Q_o$ ou :

$-\frac{1}{3}fs_oq\left(\frac{\dot{q}}{q} + \frac{\dot{s}_o}{s_o}\right)t = -\frac{1}{3}fs_oq$ ou $\frac{\dot{q}}{q} + \frac{\dot{s}_o}{s_o} = \frac{1}{t}$ et (³) devient :

$\frac{1}{3}fq^2\left[\frac{13}{35}\frac{\dot{q}}{q} + \frac{3}{14}\left(\frac{1}{t} - \frac{\dot{q}}{q}\right)\right] = k\left(\frac{7}{10}h_1 + \frac{3}{7}\cdot\frac{3\dot{Q}_o t}{fq}\right)$ ou, toutes réductions faites :

$$\dot{q} = -\frac{15}{11}\frac{q}{t} + \frac{147}{11}\frac{kh_1}{fq} + \frac{270}{11}\frac{k}{f^2q^2}\dot{Q}_o t \quad (^4)$$

Posons : $\sigma = \frac{s}{h_1}$; $\sigma_o = \frac{s_o}{h_1}$; $\lambda = \frac{-\dot{Q}_o}{kh_1}$ et $\alpha = \frac{kt}{fh_1}$, (4) devient :

$\frac{dq}{d\alpha} = -\frac{15}{11}\frac{q}{\alpha} + \frac{147}{11}\frac{h_1^2}{q} - \frac{270\lambda}{11}\frac{h_1^3\alpha}{q^2}$ ou encore, en posant $\rho = \frac{q}{h_1}$

$$\boxed{\frac{d\rho}{d\alpha} = -\frac{15}{11}\frac{\rho}{\alpha} + \frac{147}{11}\frac{1}{\rho} - \frac{270\lambda}{11}\frac{\alpha}{\rho^2}}$$

dont la solution, pour les petites valeurs de α, est $\rho = \sqrt{\frac{294}{41}\alpha}$ $\rho = 0.084680173$ pour $\alpha = 0.001$.

$s = s_o\left(1 - \frac{x}{q}\right)^2 = \frac{-3\dot{Q}_o t}{fq}\left(1 - \frac{x}{q}\right)^2$ ou $\frac{s}{\left(-\frac{\dot{Q}_o}{kh_1}\right)} = \frac{3kh_1t}{fq}\left(1 - \frac{x}{q}\right)^2$

2.

ou $\frac{\delta}{\lambda} = \frac{3h_1^2\alpha}{q}\left(1-\frac{x}{q}\right)^2$ et, en posant $\xi = \frac{x}{h_1}$ $\frac{\sigma}{\lambda} = \frac{3\alpha}{\rho}\left(1-\frac{\xi}{\rho}\right)^2$ ou, en fonction de u,

$\sigma = \sigma_0\left(1-\frac{4}{3}\frac{\sigma_0 u^2}{\lambda\xi}\right)^2$. On a $\delta_0 = -\frac{3\dot{Q}_0 t}{fq} = \frac{-3\dot{Q}_0 f h_1^2\alpha}{fq\,k\,h_1} = 3\lambda\frac{h_1^2\alpha}{q}$ ou $\sigma_0 = 3\lambda\frac{\alpha}{\rho}$

Puisque $\sigma_0 \leq 1$, on doit avoir $\frac{\rho}{\alpha} > 3\lambda$

Le problème : $(h_1-\delta)\frac{\partial^2\delta}{\partial x^2} - \left(\frac{\partial\delta}{\partial x}\right)^2 = \frac{f}{k}\frac{\partial\delta}{\partial t}$ avec $\begin{cases}\delta(\infty, t) = 0\\ \delta(x,0) = 0\\ k\left[(h_1-\delta)\frac{\partial\delta}{\partial x}\right]_{x=0} = \dot{Q}_0\end{cases}$

on demande $\delta = \delta(x,t)$

Solution : $\dot{q} = -\frac{15}{11}\frac{q}{t} + \frac{147}{11}\frac{k h_1}{fq} + \frac{270}{11}\frac{k}{f^2q^2}\dot{Q}_0 t$

$\frac{\sigma}{\lambda} = \frac{3\alpha}{\rho}\left(1-\frac{\xi}{\rho}\right)^2$ avec ρ donné par $\frac{d\rho}{d\alpha} = -\frac{15}{11}\frac{\rho}{\alpha} + \frac{147}{11}\frac{1}{\rho} - \frac{270}{11}\frac{\lambda\alpha}{\rho^2}$

$\sigma_0 = 3\lambda\frac{\alpha}{\rho}$

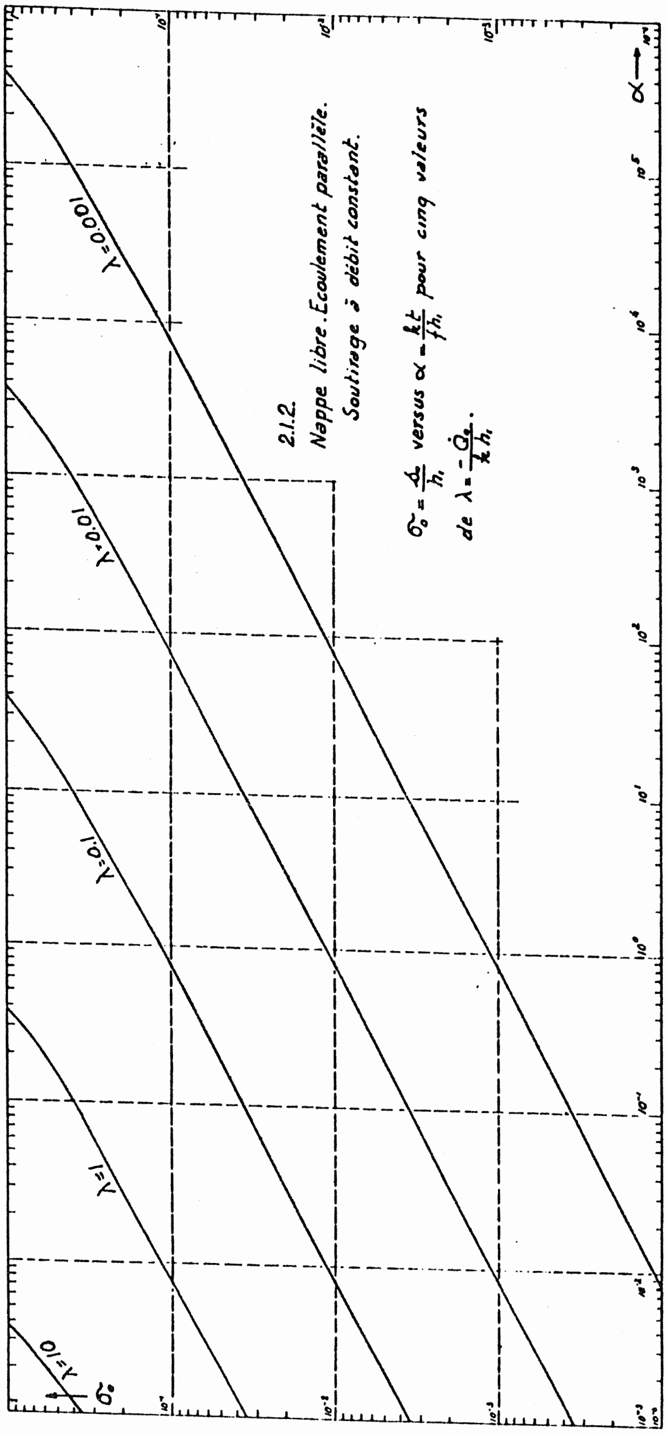
$\sigma = \frac{\delta}{h_1}$; $\sigma_0 = \frac{\delta_0}{h_1}$; $\lambda = \frac{-\dot{Q}_0}{k h_1}$; $\alpha = \frac{kt}{f h_1}$; $\rho = \frac{q}{h_1}$ et $\xi = \frac{x}{h_1}$

α	ρ				
	$\lambda = 0.001$	$\lambda = 0.01$	$\lambda = 0.1$	$\lambda = 1$	$\lambda = 10$
0.001	0.08468	0.08468	0.08468	0.08468	0.08468
15	0.10371	0.10370	0.10361	0.10270	0.092289
2	0.11975	0.11974	0.119602	0.11819	0.10072
3	0.14667	0.14664	0.14643	0.14421	0.11313
4	0.16935	0.16933	0.16904	0.16604	$\rho > 30\alpha$
5	0.18934	0.18931	0.18894	0.18518	
6	0.20741	0.20737	0.20693	0.20240	
7	0.22403	0.22398	0.22347	0.21816	
8	0.23950	0.23944	0.23886	0.23277	
9	0.25403	0.25397	0.25331	0.24644	
0.01	0.26777	0.267701	0.26697	0.25931	
15	0.32795	0.32784	0.32674	0.31513	
2	0.37868	0.37854	0.37707	0.36143	
3	0.46379	0.46357	0.46136	0.43748	
4	0.53553	0.53524	0.53229	0.49996	
5	0.59874	0.59837	0.59469	0.55370	
6	0.65588	0.65544	0.65102	0.60119	
7	0.70843	0.70792	0.70275	0.64390	
8	0.75734	0.75675	0.75084	0.68278	
9	0.80327	0.80261	0.79596	0.71852	
0.1	0.84672	0.84599	0.83859	0.75159	
15	1.03699	1.03590	1.02476	0.88727	
2	1.19739	1.19593	1.18105	0.98837	
3	1.46646	1.46427	1.44182	1.12217	
4	1.69328	1.69035	1.66031	$\rho > 3\alpha$	
5	1.89310	1.88944	1.85174		
6	2.07374	2.06935	2.02396		
7	2.23986	2.23474	2.18161		
8	2.39447	2.38861	2.32772		
9	2.53967	2.53309	2.46439		

α	$\lambda = 0.001$	$\lambda = 0.01$	$\lambda = 0.1$	$\lambda = 1$	$\lambda = 10$
	ϱ				
1	2.67701	2.66969	2.59316		
1.5	3.27843	3.26744	3.15129		
2	3.78539	3.77072	3.61428		
3	4.63569	4.61365	4.37484		
4	5.35240	5.32297	4.99964		
5	5.98374	5.94691	5.53706		
6	6.55443	6.51020	6.01192		
7	7.07917	7.02752	6.43899		
8	7.56753	7.50845	6.82786		
9	8.02616	7.95964	7.18521		
10	8.45990	8.38593	7.51589		
15	10.35899	10.24765	8.87270		
20	11.95935	11.81047	9.88367		
30	14.64268	14.41826	11.22167		
40	16.90355	16.60308	$\varrho > 0.3\alpha$		
50	18.89445	18.51746			
60	20.69356	20.23964			
70	22.34737	21.81614			
80	23.88613	23.27722			
90	25.33087	24.64395			
10^2	26.69689	25.93163			
1.5	32.67437	31.51292			
2	37.70719	36.14280			
3	46.13649	43.74840			
4	53.22976	49.99641			
5	59.46916	55.37062			
6	65.10198	60.11927			
7	70.27521	64.38991			
8	75.08453	68.27863			
9	79.59644	71.85212			
10^3	83.85935	75.15889			
1.5	102.47653	88.72705			
2	118.10476	98.83676			
3	144.18262	112.21672			
4	166.03078	$\varrho > 0.03\alpha$			
5	185.17461				
6	202.39645				
7	218.16138				
8	232.77225				
9	246.43947				
10^4	259.31630				
1.5	315.12925				
2	361.42805				
3	437.48405				
4	499.96413				
5	553.70619				
6	601.19269				
7	643.89915				
8	682.78634				
9	718.52124				
10^5	751.58889				
1.5	887.27052				
2	988.36764				
3	1122.16719				
	$\varrho > 0.003\alpha$				

α	$\sigma_0 = 3\lambda \frac{\alpha}{\rho}$				
	$\lambda = 0.001$	$\lambda = 0.01$	$\lambda = 0.1$	$\lambda = 1$	$\lambda = 10$
0.001	0.0000 35427	0.000354274	0.003542742	0.035427419	0.354274194
15	43390	433937	4343132	43815291	0.487597634
2	50103	501084	5016628	50763016	0.595679223
3	61363	613722	6146317	62409800	0.795543872
4	70856	708684	7099060	72271254	
5	79220	792351	7938820	81002376	
6	86781	867995	8698346	88933083	
7	93735	937559	9397072	96258219	
8	0.000100207	0.001002310	0.010047667	0.103104545	
9	106285	1063128	10658928	109559999	
0.01	112035	1120653	11237259	115688563	
15	137215	1372607	13772259	142798468	
2	158443	1585041	15912085	166008111	
3	194054	1941458	19507334	205721776	
4	224076	2241984	22543780	240017214	
5	250527	2506794	25223156	270901788	
6	274440	2746234	27648926	299404836	
7	296431	2966448	29882516	326138030	
8	316900	3171442	31963974	351500879	
9	336125	3363997	33921113	375771770	
0.1	354308	3546138	35774184	399154381	
15	433947	4344053	43912495	507173392	
2	501087	5016993	50802353	607061558	
3	613723	6146416	62420836	802019524	
4	708684	7099099	72275754		
5	792351	7938839	81004627		
6	867995	8698357	88934365		
7	937559	9397078	96259017		
8	0.001002310	0.010047671	0.103105075		
9	1063128	10658930	109560369		
1	1120653	11237262	115688832		
1.5	1372607	13772260	142798548		
2	1585041	15912085	166008145		
3	1941458	19507334	205721786		
4	2241984	22543780	240017218		
5	2506794	25223156	270901791		
6	2746234	27648926	299404837		
7	2966448	29882516	326138031		
8	3171442	31963974	351500880		
9	3363997	33921113	375771770		
10	3546138	35774184	399154381		
15	4344053	43912495	507173392		
20	5016993	50802353	607061558		
30	6146416	62420836	802019524		
40	7099099	72275754			
50	7938839	81004627			
60	8698357	88934365			
70	9397078	96259017			
80	0.0 10047671	0.103105075			
90	0.0 10658930	0.109560369			
10^2	0.0 11237262	0.115688832			

α	$\lambda = 0.001$	$\lambda = 0.01$
	σ_0	
$1.5.10^2$	0.013772260	0.142798548
2	15912085	.166008145
3	19507334	205721786
4	22543780	240017218
5	25223156	270901791
6	27648926	299404837
7	29882516	326138031
8	31963974	351500880
9	33921113	375771770
10^3	35774184	399154381
1.5	43912495	507173392
2	50802353	607061558
3	62420836	802019524
4	72275753	
5	81004627	
6	88934365	
7	96259017	
8	0.103105075	
9	109560369	
10^4	115688832	
1.5	142798548	
2	166008145	
3	210534171	
4	240017218	
5	270901791	
6	299404837	
7	326138031	
8	351500880	
9	375771770	
10^5	399154381	
1.5	507173392	
2	607061558	
3	802019524	


2.1.2.
Nappe libre. Ecoulement parallèle.
Soutirage à débit constant.
$\sigma_0 = \frac{\Delta}{h_1}$ versus $\alpha = \frac{kt}{f h_1}$ pour cinq valeurs
de $\lambda = \frac{-Q_e}{k h_1}$.
λ=0.001
λ=0.01
λ=0.1
λ=1
λ=10
σ_0
α

2.2.1. Nappe libre. Ecoulement axisymétrique. Soutirage à niveau constant : s_0.

Continuité : $\frac{d\dot{Q}}{dr} = 2\pi f r \dot{s}$ ou $\frac{dQ}{dr} = 2\pi f r s$

Darcy : $\dot{Q} + 2\pi k r h \frac{dh}{dr} = 0$ ou $\frac{1}{2\pi r k}\dot{Q} = (h_1 - s)\frac{ds}{dr}$ ou :

$$\frac{1}{2\pi k}\int_a^R \dot{Q}\,\delta Q\,\frac{dr}{r} = \int_a^R (h_1 - s)\frac{ds}{dr}\,\delta Q\,dr \qquad \text{avec} \quad \dot{Q} = \frac{\partial Q}{\partial R}\dot{R}, \quad \delta Q = \frac{\partial Q}{\partial R}\delta R \qquad \text{d'où}$$

$$\frac{1}{2\pi k}\int_a^R \left(\frac{\partial Q}{\partial R}\right)^2 \dot{R}\,\delta R\,\frac{dr}{r} = \int_a^R (h_1 - s)\frac{ds}{dr}\frac{\partial Q}{\partial R}\,\delta R\,dr \qquad (1')$$

Soit $s = s_0 \frac{u}{u_0}$ avec $\begin{cases} u = \log \frac{R}{r} \\ u_0 = \log \frac{R}{a} \end{cases}$ $\quad \frac{\partial Q}{\partial R} = \frac{1}{R}\frac{\partial Q}{\partial u_0}$; $\frac{dr}{r} = -du$; $\frac{ds}{dr} = -\frac{1}{r}\frac{s_0}{u_0}$ et $(1')$ devient :

$$\frac{\dot{u}_0}{2\pi k}\int_0^u \left(\frac{\partial Q}{\partial u_0}\right)^2 du = -\frac{s_0}{u_0}\int_0^u \left(h_1 - \frac{s_0}{u_0}u\right)\frac{\partial Q}{\partial u_0}\,du \qquad (2)$$

Q a déja été calculé, il suffit d'y changer S en f.

$$Q = \frac{-\pi f a^2 s_0 u_0}{2u_0^2 e^{-2u_0}}\underbrace{(1 - e^{-2u} - 2u e^{-2u})}_{\varphi} \quad \text{et} \quad \frac{\partial Q}{\partial u_0} = \frac{-\pi f a^2 s_0}{2u_0^2 e^{-2u_0}}\left[u_0\frac{\partial \varphi}{\partial u} - (1 - 2u_0)\varphi\right]$$

et $\frac{\partial Q_0}{\partial u_0} = \frac{-\pi f a^2 s_0}{2u_0^2 e^{-2u_0}}\varphi_0$ et (2) devient :

$$\frac{\dot{u}_0}{2\pi k}\int_0^{u_0}\left(\frac{\partial Q}{\partial u_0}\right)^2 du = \frac{\dot{u}_0}{2\pi k}\left(\frac{\pi f a^2}{2u_0^2 e^{-2u_0}}\right)^2 s_0^2 \underbrace{\int_0^{u_0}\left[u_0\frac{\partial\varphi}{\partial u} - (1 - 2u_0)\varphi\right]^2 du}_{\text{déja calculé et noté } I_1} \quad \text{et}$$

$$-\frac{s_0}{u_0}\int_0^{u_0}\left(h_1 - s_0\frac{u}{u_0}\right)\frac{\partial Q}{\partial u_0}\,du = \frac{\pi f a^2 s_0}{2u_0^2 e^{-2u_0}}\frac{s_0}{u_0}\int_0^{u_0}\left(h_1 - \frac{s_0}{u_0}u\right)\left[u_0\frac{\partial\varphi}{\partial u} - (1 - 2u_0)\varphi\right]du$$

$$= \frac{\pi f a^2 s_0^2}{2u_0^2 e^{-2u_0}}\frac{1}{u_0}\left[h_1 u_0 \underbrace{\int_0^{u_0}\frac{\partial\varphi}{\partial u}du}_{J_1} - h_1(1 - 2u_0)\underbrace{\int_0^{u_0}\varphi\,du}_{J_2} - s_0\underbrace{\int_0^{u_0} u\frac{\partial\varphi}{\partial u}du}_{J_3} + \frac{s_0}{u_0}(1 - 2u_0)\underbrace{\int_0^{u_0} u\,\varphi\,du}_{J_4}\right]$$

$J_1 = 1 - e^{-2u_0} - 2u_0 e^{-2u_0} = \varphi_0$

$J_2 = u_0 e^{-2u_0} + u_0 + e^{-2u_0} - 1$ noté I_5

$J_3 = 2u_0^2 e^{-2u_0} - 2u_0 e^{-2u_0} - e^{-2u_0} + 1$

$J_4 = \frac{1}{4}(4u_0^2 e^{-2u_0} + 2u_0^2 + 6u_0 e^{-2u_0} + 3e^{-2u_0} - 3)$ d'où :

$$\dot{u}_0 I_1 = \frac{4k e^{-2u_0}}{f a^2}\left\{h_1 u_0 \underbrace{\left[u_0 J_1 - (1 - 2u_0)J_2\right]}_{= I_4} - s_0 \underbrace{\left[u_0 J_3 - (1 - 2u_0)J_4\right]}_{I_7}\right\}$$

avec $I_4 = 2u_o^2 - 2u_o - e^{-2u_o} + 1$ et $I_7 = u_o^3 - \frac{1}{2}u_o^2 - u_o e^{-2u_o} - \frac{1}{2}u_o - \frac{3}{4}e^{-2u_o} + \frac{3}{4}$

$\dot{u}_o = \frac{k h_1}{f a^2} \frac{4(u_o I_4 - \frac{s_o}{h_1} I_7) e^{-2u_o}}{I_1}$. Soit $\alpha = \frac{k h_1 t}{f a^2}$ et $\sigma_o = \frac{s_o}{h_1}$, il vient :

$$\boxed{\frac{du_o}{d\alpha} = \frac{4(u_o I_4 - \sigma_o I_7)e^{-2u_o}}{I_1}}$$

Pour u_o petit $\frac{du_o}{d\alpha} = \frac{4(u_o \frac{4}{3}u_o^3 - \sigma_o \frac{5}{8}u_o^4)}{\frac{32}{15}u_o^5} = \frac{1}{u_o}\frac{5}{16}(8 - 5\sigma_o)$

ou $u_o^2 = \frac{5}{8}(8 - 5\sigma_o)\alpha$ ∴ pour $\alpha = 0.00015$ et pour
$\sigma_o = 0$ $u_o = 0.027386128$
$\sigma_o = 0.1$ $u_o = 0.026516504$
$\sigma_o = 1/3$ $u_o = 0.024366986$
$\sigma_o = 2/3$ $u_o = 0.020916501$
$\sigma_o = 1$ $u_o = 0.016770510$

$$\dot{Q}_o = \left(\frac{\partial Q}{\partial u_o}\right)_o \dot{u}_o = \frac{-\pi f a^2 s_o}{2u_o^2 e^{-2u_o}} \frac{k h_1}{f a^2} \frac{4(u_o I_4 - \sigma_o I_7)e^{-2u_o}}{I_1}.$$

ou $$\boxed{\frac{\dot{Q}_o}{(2\pi k h_1) s_o} = \frac{(u_o I_4 - \sigma_o I_7)\,\Psi_o}{u_o^2 I_1}}$$

Pour $\sigma_o = 0$ ($h_1 = \infty$), on retrouve l'approximation retenue pour les nappes confinées.

Le problème : $$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} - \frac{1}{h_1 - s}\left(\frac{\partial s}{\partial r}\right)^2 = \frac{f}{k}\frac{1}{h_1 - s}\frac{\partial s}{\partial t}$$

avec $s(0,t) = s_o$; $s(\infty,t) = 0$; $s(r,0) = 0$

On demande : $s = s(r,t)$ et $\dot{Q}_o = 2\pi k \left[r(h_1 - s)\frac{\partial s}{\partial r}\right]_{r=a}$

Solution : $\sigma = \frac{s}{h_1} = \frac{s_o}{h_1}\left(1 - \frac{1}{u_o}\log\frac{r}{a}\right)$ et $\frac{\dot{Q}_o}{(2\pi k h_1)s_o} = \frac{(u_o I_4 - \sigma_o I_7)\,\Psi_o}{u_o^2 I_1}$

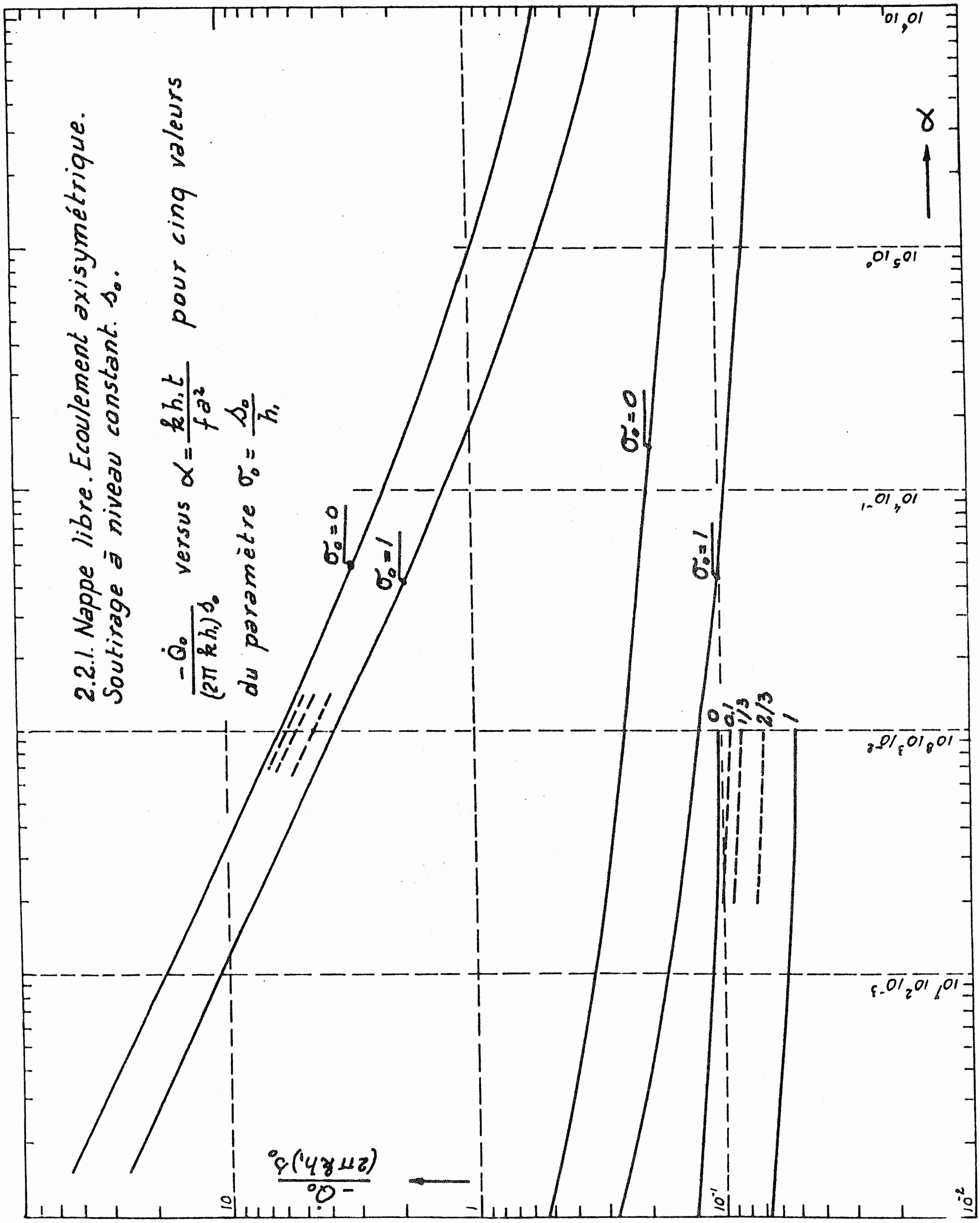
avec $\frac{du_o}{d\alpha} = \frac{4(u_o I_4 - \sigma_o I_7)e^{-2u_o}}{I_1}$; $\alpha = \frac{k h_1 t}{f a^2}$

2.

$$"\dot{Q}_0" = \frac{\dot{Q}_0}{(2\pi k h_1)\vartheta_0}$$

	$\sigma_0 = 0$		$\sigma_0 = 0.1$		$\sigma_0 = 1/3$		$\sigma_0 = 2/3$		$\sigma_0 = 1$	
α	u_0	$"\dot{Q}_0"$	u_0	$"\dot{Q}_0"$	u_0	$"\dot{Q}_0"$	u_0	$"\dot{Q}_0"$	u_0	$"\dot{Q}_0"$
$1.5 \cdot 10^{-4}$	0.02738	46.8721	0.02651	44.3927	0.02436	38.0924	0.02091	32.3241	0.01677	25.9024
2	0.03146	39.3509	0.03046	38.4996	0.02798	34.8049	0.02410	28.3873	0.01942	23.2617
3	0.03829	32.4357	0.03710	31.5051	0.03404	28.4063	0.02930	25.2376	0.02358	18.8395
4	0.04402	28.2305	0.04265	27.3499	0.03917	25.1422	0.03373	21.5528	0.02713	17.5362
5	0.04904	25.3968	0.04752	24.5414	0.04367	22.5535	0.03762	19.2829	0.03026	15.6119
6	0.05357	23.2268	0.05192	22.4537	0.04773	20.6764	0.04113	17.6518	0.03309	14.1208
7	0.05772	21.5388	0.05594	20.8529	0.05145	19.1668	0.04435	16.4124	0.03569	13.1372
8	0.06157	20.1932	0.05968	19.5416	0.05490	17.9681	0.04734	15.3411	0.03811	12.2797
9	0.06518	19.0846	0.06317	18.4600	0.058139	16.9474	0.050135	14.5213	0.040379	11.6303
10^{-3}	0.06858	18.11564	0.06647	17.5406	0.061188	16.1116	0.052774	13.7905	0.042519	11.0406
1.5	0.08334	14.89606	0.08080	14.4109	0.074447	13.2234	0.064261	11.3131	0.051847	9.0601
2	0.09563	12.96813	0.09274	12.5454	0.085500	11.5075	0.073857	9.8455	0.059654	7.8637
3	0.11594	10.67903	0.11247	10.3284	0.103790	9.4686	0.089770	8.0928	0.072634	6.4612
4	0.13277	9.31304	0.12882	9.0059	0.118969	8.2524	0.103007	7.0496	0.083463	5.6235
5	0.14737	8.38040	0.14302	8.1029	0.132165	7.4220	0.114537	6.3370	0.092917	5.0520
6	0.16040	7.69159	0.15570	7.4359	0.143956	6.8089	0.124856	5.8108	0.101397	4.6302
7	0.17224	7.15604	0.16722	6.9175	0.154683	6.3323	0.134259	5.4019	0.109140	4.3022
8	0.18315	6.72423	0.17784	6.4994	0.164571	5.9480	0.142939	5.0721	0.116298	4.0378
9	0.19329	6.36645	0.18771	6.1530	0.173774	5.6296	0.151027	4.7989	0.122981	3.8188
10^{-2}	0.20278	6.06372	0.19696	5.8599	0.182406	5.3603	0.158622	4.5678	0.129264	3.6335
1.5	0.24339	5.03605	0.23654	4.8650	0.219395	4.4460	0.191265	3.7834	0.156375	3.0047
2	0.27643	4.42283	0.26878	4.2713	0.249596	3.9006	0.218031	3.3155	0.178729	2.6296
3	0.32959	3.69451	0.32070	3.5663	0.298379	3.2529	0.261476	2.7599	0.215247	2.1845
4	0.37237	3.25963	0.36254	3.1454	0.337801	2.8664	0.296771	2.4284	0.245123	1.9190
5	0.40864	2.96241	0.39804	2.8577	0.371334	2.6022	0.326920	2.2020	0.270789	1.7377
6	0.44036	2.74269	0.42911	2.6450	0.400744	2.4070	0.353455	2.0346	0.293488	1.6037
7	0.46868	2.57169	0.45687	2.4795	0.427070	2.2550	0.377280	1.9045	0.313954	1.4995
8	0.49435	2.43367	0.48206	2.3459	0.450984	2.1325	0.398981	1.7994	0.332665	1.4155
9	0.51789	2.31920	0.50516	2.2352	0.472950	2.0308	0.418962	1.7124	0.349951	1.3458
10^{-1}	0.53967	2.22224	0.52653	2.1414	0.493303	1.9447	0.437517	1.6386	0.366052	1.2868
1.5	0.62987	1.89225	0.61520	1.8221	0.577966	1.6518	0.515103	1.3879	0.433878	1.0864
2	0.70013	1.69444	0.68436	1.6308	0.644266	1.4764	0.576295	1.2379	0.487921	0.9666
3	0.80797	1.45818	0.79069	1.4023	0.746609	1.2671	0.671454	1.0590	0.572872	0.8239
4	0.89067	1.31608	0.87234	1.2649	0.825516	1.1414	0.745361	0.9517	0.639572	0.7384
5	0.95826	1.21835	0.93915	1.1705	0.890261	1.0549	0.806330	0.8780	0.695042	0.6797
6	1.01565	1.14570	0.99593	1.1003	0.945411	0.9908	0.858482	0.8233	0.742794	0.6363
7	1.06565	1.08886	1.04543	1.0453	0.993583	0.9406	0.904192	0.7806	0.784867	0.6023
8	1.11004	1.04276	1.08940	1.0008	1.036431	0.8999	0.944965	0.7460	0.822564	0.5749
9	1.15000	1.00435	1.12899	0.9637	1.075068	0.8660	0.981823	0.7172	0.856770	0.5521
10^{0}	1.18636	0.97169	1.16505	0.9322	1.110286	0.8372	1.015490	0.6928	0.888120	0.5327
1.5	1.33154	0.85936	1.30911	0.8238	1.251363	0.7384	1.150992	0.6089	1.015242	0.4664
2	1.43927	0.79093	1.41612	0.7578	1.356486	0.6783	1.252563	0.55810	1.111440	0.4263
3	1.59710	0.70772	1.57308	0.6775	1.511094	0.6053	1.402756	0.4965	1.254939	0.3778
4	1.71294	0.65665	1.688381	0.6283	1.624946	0.5606	1.513893	0.4589	1.361966	0.3483
5	1.80478	0.62096	1.779844	0.5939	1.715409	0.5294	1.602482	0.4327	1.447729	0.3279
6	1.88100	0.59405	1.855790	0.5680	1.790613	0.5060	1.676298	0.4130	1.519465	0.3125
7	1.94622	0.57276	1.920798	0.5476	1.855043	0.4874	1.739653	0.3974	1.581216	0.3004
8	2.00327	0.55531	1.977666	0.5308	1.911447	0.4722	1.795194	0.3848	1.635479	0.2905
9	2.05398	0.54064	2.028235	0.5166	1.961633	0.4595	1.844669	0.3741	1.68391	0.2822

α	u_0 ($\sigma_0=0$)	"$\dot{Q}_0$" ($\sigma_0=0$)	u_0 ($\sigma_0=0.1$)	"$\dot{Q}_0$" ($\sigma_0=0.1$)	u_0 ($\sigma_0=1/3$)	"$\dot{Q}_0$" ($\sigma_0=1/3$)	u_0 ($\sigma_0=2/3$)	"$\dot{Q}_0$" ($\sigma_0=2/3$)	u_0 ($\sigma_0=1$)	"$\dot{Q}_0$" ($\sigma_0=1$)
10	2.0996	0.5280	2.0737	0.5045	2.0068	0.4486	1.8893	0.3650	1.7276	0.2752
1.5	2.2778	0.4839	2.2515	0.4622	2.1835	0.4103	2.0640	0.3331	1.8995	0.2505
2	2.4063	0.4563	2.3798	0.4356	2.3112	0.3864	2.1905	0.3132	2.0245	0.2352
3	2.5901	0.4218	2.5633	0.4025	2.4940	0.3566	2.3721	0.2885	2.2044	0.2162
4	2.7220	0.4000	2.6951	0.3816	2.6254	0.3378	2.5028	0.2730	2.3343	0.2043
5	2.8252	0.3844	2.7981	0.3666	2.7282	0.3244	2.6052	0.2619	2.4362	0.1959
6	2.9099	0.3725	2.8828	0.3552	2.8127	0.3142	2.6894	0.2535	2.5202	0.1894
7	2.9819	0.3629	2.9547	0.3460	2.8844	0.3059	2.7609	0.2467	2.5915	0.1843
8	3.0444	0.3550	3.0172	0.3384	2.9468	0.2991	2.8232	0.2411	2.6537	0.1800
9	3.0997	0.3483	3.0725	0.3319	3.0020	0.2933	2.8783	0.2364	2.7087	0.1764
10^2	3.1493	0.3424	3.1221	0.3264	3.0515	0.2884	2.9277	0.2323	2.7580	0.1733
1.5	3.3412	0.3216	3.3138	0.3064	3.2431	0.2705	3.1189	0.2177	2.9493	0.1622
2	3.4781	0.3081	3.4507	0.2936	3.3798	0.2591	3.2556	0.2083	3.0860	0.1551
3	3.6720	0.2909	3.6446	0.2771	3.5737	0.2444	3.4494	0.1963	3.2801	0.1461
4	3.8102	0.2797	3.7828	0.2664	3.7118	0.2349	3.5876	0.1886	3.4185	0.1403
5	3.9177	0.2716	3.8903	0.2586	3.8193	0.2280	3.6952	0.1830	3.5263	0.1361
6	4.0058	0.2653	3.9783	0.2526	3.9074	0.2226	3.7832	0.1786	3.6146	0.1328
7	4.0803	0.2602	4.0529	0.2477	3.9819	0.2183	3.8578	0.1751	3.6894	0.1302
8	4.1450	0.2559	4.1175	0.2436	4.0466	0.2146	3.9226	0.1721	3.7543	0.1280
9	4.2021	0.2523	4.1746	0.2402	4.1037	0.2115	3.9797	0.1696	3.8116	0.1261
10^3	4.2532	0.2491	4.2258	0.2371	4.1548	0.2088	4.0309	0.1674	3.8629	0.1244
1.5	4.4503	0.2375	4.4229	0.2260	4.3520	0.1990	4.2283	0.1595	4.0608	0.1185
2	4.5905	0.2299	4.5631	0.2187	4.4923	0.1925	4.3686	0.1542	4.2016	0.1146
3	4.7885	0.2199	4.7611	0.2092	4.6904	0.1841	4.5669	0.1474	4.4006	0.1095
4	4.9293	0.2133	4.9019	0.2029	4.8312	0.1785	4.7080	0.1429	4.5419	0.1061
5	5.0386	0.2085	5.0112	0.1983	4.9405	0.1744	4.8175	0.1396	4.6518	0.1037
6	5.1280	0.2047	5.1006	0.1947	5.0300	0.1712	4.9070	0.1370	4.7416	0.1018
7	5.2036	0.2015	5.1763	0.1917	5.1057	0.1686	4.9828	0.1349	4.8176	0.1002
8	5.2692	0.1989	5.2418	0.1892	5.1713	0.1664	5.0485	0.1332	4.8835	0.0989
9	5.3270	0.1967	5.2997	0.1871	5.2291	0.1645	5.1064	0.1316	4.9416	0.0977
10^4	5.3788	0.1947	5.3515	0.1852	5.2809	0.1628	5.1583	0.1310	4.9936	0.0967
5	6.1718	0.1687	6.1446	0.1604	6.0744	0.1409	5.9527	0.1127	5.7901	0.0837
10^5	6.5143	0.1595	6.4871	0.1517	6.4171	0.1332	6.2958	0.1065	6.1340	0.0791
5	7.3112	0.1415	7.2841	0.1345	7.2145	0.1181	7.0939	0.0944	6.9339	0.0701
10^6	7.6549	0.1350	7.6279	0.1283	7.5584	0.1126	7.4382	0.0900	7.2788	0.0669
5	8.4541	0.1218	8.4271	0.1158	8.3579	0.1016	8.2384	0.0812	8.0803	0.0603
10^7	8.7986	0.1169	8.7717	0.1111	8.7026	0.0975	8.5833	0.0779	8.4257	0.0579
5	9.5991	0.1069	9.5723	0.1016	9.5034	0.0892	9.3847	0.0712	9.2283	0.0529
10^8	9.9441	0.1031	9.9173	0.0980	9.8485	0.0860	9.7301	0.0686	9.5740	0.0511
5	10.7456	0.0952	10.7188	0.0905	10.6503	0.0794	10.5323	0.0634	10.37710	0.0472
10^9	11.0909	0.0922	11.0642	0.0876	10.9957	0.0769	10.8779	0.0614	10.7230	0.0457
5	11.8930	0.0858	11.8664	0.0816	11.7981	0.0716	11.6806	0.0517	11.5265	0.0425
10^{10}	12.2386	0.0834	12.2120	0.0792	12.1438	0.0695	12.0264	0.0555	11.8726	0.0413
5	13.0412	0.0781	13.0146	0.0742	12.9466	0.0651	12.8296	0.0520	12.6763	0.0387
10^{11}	13.3869	0.0761	13.3604	0.0723	13.2923	0.0634	13.1755	0.0506	13.0225	0.0377
5	14.1899	0.0717	14.1634	0.0681	14.0955	0.0597	13.9789	0.0477	13.8264	0.0356
10^{12}	14.5358	0.0700	14.5093	0.0665	14.4414	0.0583	14.3250	0.0466	14.1727	0.0347


2.2.1. Nappe libre. Ecoulement axisymétrique.
Soutirage à niveau constant. s_0.
$\frac{-\dot{Q}_0}{(2\pi k h_1) s_0}$ versus $\alpha = \frac{k h_1 t}{f a^2}$ pour cinq valeurs
du paramètre $\sigma_0 = \frac{s_0}{h_1}$
$\sigma_0 = 0$
$\sigma_0 = 1$
$\sigma_0 = 0$
$\sigma_0 = 1$
0
0.1
1/3
2/3
1
α
$\frac{-\dot{Q}_0}{(2\pi k h_1) s_0}$
10
1
10^{-1}
10^{-2}

2.2.2. Nappe libre. Ecoulement axisymétrique - Soutirage à débit constant : $-\dot{Q}_0$.

Continuité : $\frac{d\dot{Q}}{dr} = 2\pi r f \dot{s}$ ou $\frac{dQ}{dr} = 2\pi r f s$

Darcy : $\dot{Q} = 2\pi k r h \frac{dh}{dr}$ ou $\frac{1}{2\pi k}\frac{\dot{Q}}{r} - (h_1 - s)\frac{ds}{dr} = 0$ ou

$$\frac{1}{2\pi k}\int_a^R \dot{Q}\,\delta Q\,\frac{dr}{r} - \int_a^R (h_1 - s)\frac{ds}{dr}\,\delta Q\,dr = 0 \text{ avec } \begin{cases} \dot{Q} = \frac{\partial Q}{\partial R}\dot{R} + \frac{\partial Q}{\partial s_0}\dot{s}_0 \\ \delta Q = \frac{\partial Q}{\partial R}\delta R + \frac{\partial Q}{\partial s_0}\delta s_0 \end{cases} \text{ d'où}$$

$$\frac{1}{2\pi k}\int_a^R \dot{Q}\,\frac{\partial Q}{\partial R}\,\frac{dr}{r} = \int_a^R (h_1 - s)\frac{ds}{dr}\,\frac{\partial Q}{\partial R}\,dr \quad (1)$$

Soit $s = s_0 \frac{u}{u_0}$ avec $\begin{cases} u = \log\frac{R}{r} \\ u_0 = \log\frac{R}{a} \end{cases}$ $\begin{cases} r = Re^{-u} \;;\; r\,dr = -a^2\frac{e^{-2u}}{e^{-2u_0}}du \\ dr = -Re^{-u}du \;;\; \frac{dr}{r} = -du \end{cases}$

$$Q = -2\pi f\int_r^R r s\,dr = 2\pi f a^2\frac{s_0}{u_0 e^{-2u_0}}\int_u^0 u e^{-2u}du = -2\pi f\frac{a^2 s_0}{u_0 e^{-2u_0}}\int_0^u u e^{-2u}du$$

$$= \frac{-\pi f a^2 s_0 u_0}{2u_0^2 e^{-2u_0}}\left(1 - e^{-2u} - 2ue^{-2u}\right) \text{ d'où } \begin{cases} \frac{\partial Q}{\partial u_0} = \frac{-\pi f a^2 s_0}{2u_0^2 e^{-2u_0}}\left[u_0\frac{\partial\varphi}{\partial u} - (1-2u_0)\varphi\right] \\ \frac{\partial Q}{\partial s_0} = \frac{-\pi f a^2 u_0}{2u_0^2 e^{-2u_0}}\left(1 - e^{-2u} - 2ue^{-2u}\right) \end{cases}$$

$$\dot{Q} = \frac{-\pi f a^2}{2u_0^2 e^{-2u_0}}\left\{s_0\left[u_0\frac{\partial\varphi}{\partial u} - (1-2u_0)\varphi\right]\dot{u}_0 + u_0\varphi\,\dot{s}_0\right\}$$

$$\dot{Q}\frac{\partial Q}{\partial u_0} = \left(-\frac{\pi f a^2}{2u_0^2 e^{-2u_0}}\right)^2 u_0\varphi\,s_0 u_0\left\{\left[u_0\frac{\partial\varphi}{\partial u} - (1-2u_0)\varphi\right]\frac{\dot{u}_0}{u_0} + \varphi\frac{\dot{s}_0}{s_0}\right\} \text{ ; (1) devient:}$$

$$\frac{1}{2\pi k}\int_0^{u_0}\dot{Q}\,\frac{\partial Q}{\partial u_0}\,du = -\frac{s_0}{u_0^2}\int_0^{u_0}(h_1 u_0 - s_0 u)\frac{\partial Q}{\partial u_0}\,du \quad (2) \text{ or:}$$

$$\int_0^{u_0}\dot{Q}\,\frac{\partial Q}{\partial u_0}\,du = \left(-\frac{\pi f a^2}{2u_0^2 e^{-2u_0}}\right)^2 u_0 s_0^2\left\{\frac{\dot{u}_0}{u_0}\underbrace{\int_0^{u_0}\left[u_0\frac{\partial\varphi}{\partial u} - (1-2u_0)\varphi\right]^2 du}_{I_1} + \frac{\dot{s}_0}{s_0}\underbrace{\int_0^{u_0}\varphi\left[u_0\frac{\partial\varphi}{\partial u} - (1-2u_0)\varphi\right]du}_{I_2}\right\}$$

$$\text{et} \int_0^{u_0}(h_1 u_0 - s_0 u)\frac{\partial Q}{\partial u_0}\,du = \frac{-\pi f a^2 s_0}{2u_0^2 e^{-2u_0}}\underbrace{\int_0^{u_0}(h_1 u_0 - s_0 u)\left[u_0\frac{\partial\varphi}{\partial u} - (1-2u_0)\varphi\right]du}_{J_1}$$

$$J_1 = h_1 u_0^2\underbrace{\int_0^{u_0}\frac{\partial\varphi}{\partial u}\,du}_{\Sigma_1} - h_1 u_0(1-2u_0)\underbrace{\int_0^{u_0}\varphi\,du}_{\Sigma_2} - s_0 u_0\underbrace{\int_0^{u_0}u\frac{\partial\varphi}{\partial u}\,du}_{\Sigma_3} + s_0(1-2u_0)\underbrace{\int_0^{u_0}u\varphi\,d\varphi}_{\Sigma_4}$$

$\delta_1 = 1 - e^{-2u_0} - 2u_0 e^{-2u_0}$ $\qquad$ $\delta_3 = -2u_0^2 e^{-2u_0} - 2u_0 e^{-2u_0} - e^{-2u_0} + 1$

$\delta_2 = u_0 e^{-2u_0} + u_0 + e^{-2u_0} - 1 = I_5$ $\qquad$ $\delta_4 = u_0^2 e^{-2u_0} + \frac{1}{2} u_0^2 + \frac{3}{2} u_0 e^{-2u_0} + \frac{3}{4} e^{-2u_0} - \frac{3}{4}$

Toutes réductions faites on a :

$J_1 = h_1 u_0 \left(2u_0^2 - 2u_0 - e^{-2u_0} + 1\right) - s_0\left(u_0^3 - \frac{1}{2} u_0^2 - u_0 e^{-2u_0} - \frac{1}{2} u_0 - \frac{3}{4} e^{-2u_0} + \frac{3}{4}\right)$ ou

$J_1 = h_1 u_0 I_4 - s_0 I_7$ et (2) devient :

$$\frac{1}{2\pi k}\left(\frac{-\pi f a^2}{2u_0^2 e^{-2u_0}}\right)^2 u_0 s_0^2 \left(\frac{\dot u_0}{u_0} I_1 + \frac{\dot s_0}{s_0} I_2\right) = -\frac{s_0}{u_0^2}\left(\frac{-\pi f a^2}{2u_0^2 e^{-2u_0}}\right) s_0 u_0 J_1 \quad \text{ou}$$

$$\frac{fa^2}{4k}\,\frac{u_0}{e^{-2u_0}}\left(\frac{\dot u_0}{u_0} I_1 + \frac{\dot s_0}{s_0} I_2\right) = h_1 u_0 I_4 - s_0 I_7$$

$\dot Q_0 t = Q_0$ devient : $\frac{\dot s_0}{s_0} = \frac{1}{t} - \frac{\psi_0}{\varphi_0}\,\frac{\dot u_0}{u_0}$ et $s_0 = \frac{-2u_0 e^{-2u_0}\dot Q_0 t}{\pi f a^2 \varphi_0}$ d'où

$$\frac{fa^2}{4k}\,\frac{u_0}{e^{-2u_0}}\left[\frac{I_1\varphi - I_2\psi}{\varphi}\,\frac{\dot u_0}{u_0} + \frac{I_2}{t}\right] = h_1 u_0 I_4 + \frac{2u_0 e^{-2u_0} t I_7 \dot Q_0}{\pi f a^2 \varphi_0} \quad \text{ou}$$

ou $\frac{I_1\varphi - I_2\psi}{\varphi}\,\frac{\dot u_0}{u_0} + \frac{I_2}{t} = \frac{4kh_1 e^{-2u_0}}{fa^2}\left[I_4 + \frac{2e^{-2u_0} I_7 t \dot Q_0}{\pi f a^2 h_1 \varphi_0}\right]$

Posons $\alpha = \frac{4kh_1 t}{fa^2}$ d'où $\frac{F}{\varphi_0}\,\frac{du_0}{d\alpha} + \frac{I_2}{\alpha} = e^{-2u_0}\left[I_4 + \frac{2e^{-2u_0} I_7 \dot Q_0 \alpha}{\varphi_0\, 4\pi k h_1^2}\right]$. Posons $\frac{-\dot Q_0}{4\pi k h_1^2} = \lambda$

$$\boxed{\frac{du_0}{d\alpha} = \frac{\varphi}{F\alpha}\left[e^{-2u_0} I_4 \alpha - I_2 - \frac{2e^{-4u_0} I_7 \lambda}{\varphi_0}\alpha^2\right]}$$

$\dot Q_0 t = Q_0$ devient :

$s_0 = \frac{-2u_0 e^{-2u_0}\dot Q_0 \alpha}{4kh_1 \pi \varphi_0}$ ou $\boxed{\frac{s_0}{\left(-\frac{\dot Q_0}{4\pi k h_1}\right)} = \frac{2u_0 e^{-2u_0}}{\varphi_0}\alpha}$ et $\boxed{s = s_0\left(1 - \frac{1}{u_0}\log\frac{r}{a}\right)}$

Pour u_0 petit $\frac{du_0}{d\alpha} = \frac{2u_0^2}{\frac{28}{15}u_0^6\alpha}\left[\frac{4}{3} u_0^3\alpha - \frac{6}{5} u_0^5 - \frac{2\frac{5}{6}u_0^4\lambda}{2u_0^2}\alpha^2\right]$ ou $\frac{1}{2}\frac{du_0^2}{d\alpha} = -\frac{9}{7} u_0^2 + \frac{10}{7}\alpha$

dont la solution est $u_0^2 = \frac{4}{5}\alpha$ $\therefore$ Si $\alpha = 10^{-3}$, $u_0 = 0.028284271$.

Le problème : $\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} - \frac{1}{(h_1 - s)}\left(\frac{\partial s}{\partial r}\right)^2 = \frac{f}{k}\,\frac{1}{(h_1 - s)}\,\frac{\partial s}{\partial t}$ avec :

$s(\infty, t) = 0$; $s(r, 0) = 0$; $\dot Q_0 = 2\pi k a (h_1 - s_0)\left(\frac{\partial s}{\partial r}\right)_{r=a}$

Solution : $s = s_0\left(1 - \frac{1}{u_0}\log\frac{r}{a}\right)$ avec $\frac{s_0}{-\frac{\dot Q_0}{4\pi k h_1}} = \frac{2u_0 e^{-2u_0}}{\varphi_0}\alpha$

et $\frac{du_0}{d\alpha} = \frac{e^{-2u_0} I_4 \alpha - I_2 - \frac{2e^{-4u_0} I_7 \lambda}{\varphi_0}\alpha^2}{F\alpha}$

$\alpha = \frac{4kh_1 t}{fa^2}$ et $\lambda = \frac{-\dot Q_0}{4\pi k h_1^2}$

2.

α	$\lambda = 0.01$		$\lambda = 0.05$		$\lambda = 0.1$		$\lambda = 1$		$\lambda = 10$	
	u_0	"s_0"	u_0	"s_0"	u_0	"s_0"	u_0	"s_0"	u_0	"s_0"
10^{-3}	0.0282	0.03469	0.0282	0.03469	0.0282	0.03469	0.0282	0.03469	0.0282	0.03469
1.5	0.0347	0.04221	0.0353	0.04152	0.0344	0.04256	0.0342	0.04285	0.0310	0.04739
2	0.0395	0.04919	0.0397	0.04896	0.0394	0.04932	0.0392	0.04968	0.0338	0.05780
3	0.0480	0.06040	0.0481	0.06037	0.0490	0.06047	0.0473	0.06141	0.0374	0.07804
4	0.0552	0.06974	0.0552	0.06980	0.0552	0.06982	0.0542	0.07113	0.0392	0.09925
5	0.0616	0.07789	0.0615	0.07796	0.0614	0.07804	0.0603	0.07964		
6	0.0672	0.08522	0.0672	0.08530	0.0671	0.08539	0.0657	0.08733		
7	0.0725	0.09194	0.0724	0.09203	0.0723	0.09214	0.0706	0.09441		
8	0.0773	0.09817	0.0772	0.09828	0.0771	0.09841	0.0752	0.10101		
9	0.0818	0.10401	0.08177	0.10414	0.0816	0.10428	0.0795	0.10722		
10^{-2}	0.0861	0.10952	0.0860	0.10966	0.0859	0.10983	0.0835	0.11310		
1.5	0.1046	0.13354	0.1045	0.13374	0.1043	0.13399	0.1008	0.13897		
2	0.1200	0.15361	0.1198	0.15388	0.1195	0.15421	0.1149	0.16095		
3	0.1453	0.18695	0.1450	0.18735	0.1446	0.18785	0.1378	0.19817		
4	0.1662	0.21473	0.1658	0.21526	0.1654	0.21593	0.1563	0.22996		
5	0.1843	0.23897	0.1838	0.23963	0.1833	0.24046	0.1720	0.25831		
6	0.2004	0.26070	0.2000	0.26148	0.1992	0.26248	0.1857	0.28425		
7	0.2151	0.28052	0.2144	0.28143	0.2137	0.28259	0.1980	0.30840		
8	0.2285	0.29884	0.2278	0.29988	0.2269	0.30120	0.2091	0.33113		
9	0.2410	0.31593	0.2402	0.31709	0.2393	0.31858	0.2192	0.35274		
10^{-1}	0.2526	0.33199	0.2518	0.33328	0.2508	0.33493	0.2286	0.37342		
1.5	0.3024	0.40121	0.3012	0.40312	0.2997	0.40557	0.2665	0.46725		
2	0.3427	0.45819	0.3412	0.46070	0.3393	0.46394	0.2949	0.55155		
3	0.4071	0.55107	0.4051	0.55477	0.4024	0.55956	0.3343	0.70883		
4	0.4587	0.62688	0.4561	0.63174	0.4527	0.63807	0.3581	0.86677		
5	0.5022	0.69189	0.4991	0.69788	0.4951	0.70571				
6	0.5400	0.74930	0.5365	0.75640	0.5319	0.76571				
7	0.5737	0.80099	0.5697	0.80917	0.5646	0.81996				
8	0.6042	0.84818	0.5998	0.85744	0.5941	0.86967				
9	0.6320	0.89173	0.6272	0.90204	0.6210	0.91572				
1	0.65774	0.93226	0.6526	0.94360	0.6458	0.95870				
1.5	0.7635	1.10252	0.7567	1.11886	0.7477	1.14089				
2	0.8452	1.23768	0.8370	1.25874	0.8261	1.28746				
3	0.9696	1.44926	0.9591	1.47915	0.9449	1.52066				
4	1.0641	1.61464	1.0517	1.65273	1.0347	1.70644				
5	1.1409	1.75168	1.1268	1.79748	1.1073	1.86292				
6	1.2058	1.86930	1.1902	1.92240	1.1685	1.99918				
7	1.2621	1.97266	1.2452	2.03273	1.2215	2.12052				
8	1.3119	2.06505	1.2938	2.13179	1.2682	2.23031				
9	1.3566	2.14870	1.3374	2.22186	1.3100	2.33086				
10	1.3971	2.22523	1.3769	2.30458	1.3479	2.42383				

$$"s_0" = \frac{s_0}{\left(-\frac{\dot{Q}_0}{4\pi k h_1}\right)} = \frac{s_0/h_1}{\left(\frac{-\dot{Q}_0}{4\pi k_1 h_1^2}\right)} = \frac{\sigma_0}{\lambda} \; ; \quad \sigma_0 \leq 1 \text{ d'où } "s_0" \leq \frac{1}{\lambda}$$

10	1.3971	2.22523	1.3769	2.30458	1.3479	2.42383
1.5	1.5581	2.53411	1.5336	2.64168	1.4975	2.80950
2	1.6766	2.76634	1.6438	2.89374	1.6067	3.11178
3	1.8489	3.11042	1.8161	3.28576	1.7643	3.58256
4	1.9745	3.36524	1.9377	3.57748	1.8779	3.95210
5	2.0736	3.56838	2.0336	3.81334	1.9667	4.26179
6	2.1555	3.73763	2.1128	4.01219	2.0395	4.53162
7	2.2253	3.88280	2.1803	4.18455	2.1011	4.77287
8	2.2863	4.01000	2.2391	4.33697	2.1544	4.99261
9	2.3403	4.12325	2.2913	4.47380	2.2013	5.19558
10^2	2.3889	4.22534	2.3381	4.59810	2.2431	5.3851
1.5	2.5777	4.62459	2.5200	5.09314	2.4021	6.2005
2	2.7131	4.91315	2.6502	5.46050	2.5112	6.8913
3	2.9058	5.32621	2.8351	6.00157	2.6539	8.1397
4	3.0435	5.62301	2.9670	6.40253	2.7378	9.4275
5	3.1507	5.85511	3.0695	6.72381		
6	3.2387	6.04597	3.1535	6.99335		
7	3.3132	6.20794	3.2245	7.22629		
8	3.3779	6.34872	3.2861	7.43199		
9	3.4350	6.47326	3.3405	7.61659		
10^3	3.4862	6.58494	3.3891	7.78430		
1.5	3.6836	7.01712	3.5762	8.45430		
2	3.8240	7.32559	3.7087	8.95560		
3	4.0224	7.76313	3.8951	9.70495		
4	4.1635	8.07504	4.0268	10.27202		
5	4.2730	8.31794	4.1285	10.73530		
6	4.3625	8.51740	4.2113	11.13273		
7	4.4383	8.68598	4.2810	11.48239		
8	4.5040	8.83229	4.3411	11.79704		
9	4.5620	8.96158	4.3939	12.08465		
10^4	4.6138	9.07743	4.4410	12.35078		
1.5	4.8135	9.52517	4.6199	13.47013		
2	4.9554	9.84387	4.7440	14.38700		
3	5.1553	10.29569	4.9119	15.96602		
4	5.2974	10.61716	5.0219	17.46586		
5	5.4075	10.86746	5.0954	19.12063		
6	5.4975	11.07365				
7	5.5737	11.24704				
8	5.6397	11.39755				
9	5.6979	11.53063				
10^5	5.7500	11.64988				
1.5	5.9504	12.11126				
2	6.0927	12.43919				
3	6.2933	12.90484				
4	6.4356	13.23564				
5	6.5461	13.49341				
6	6.6362	13.70715				
7	6.7125	13.88519				
8	6.7787	14.03996				
9	6.8370	14.17693				
10^6	6.8892	14.29976				

$\lambda = 0.01$ (suite)

α	u_0	"S_0"
10^6	6.8892	14.29975
10^7	8.0295	17.03369
10^8	9.1705	19.8605
10^9	10.3116	22.73078
10^{10}	11.4527	25.83692
10^{11}	12.5935	29.01406
10^{12}	13.7338	32.34083
10^{13}	14.8736	35.84070
10^{14}	16.0126	39.54393
10^{15}	17.1507	43.49076
10^{16}	18.2875	47.73673
10^{17}	19.4226	52.36242
10^{18}	20.5555	57.49292
10^{19}	21.6851	63.34254
10^{20}	22.8093	70.34095
10^{21}	23.9222	79.66270
8.10^{21}	24.8784	97.90774

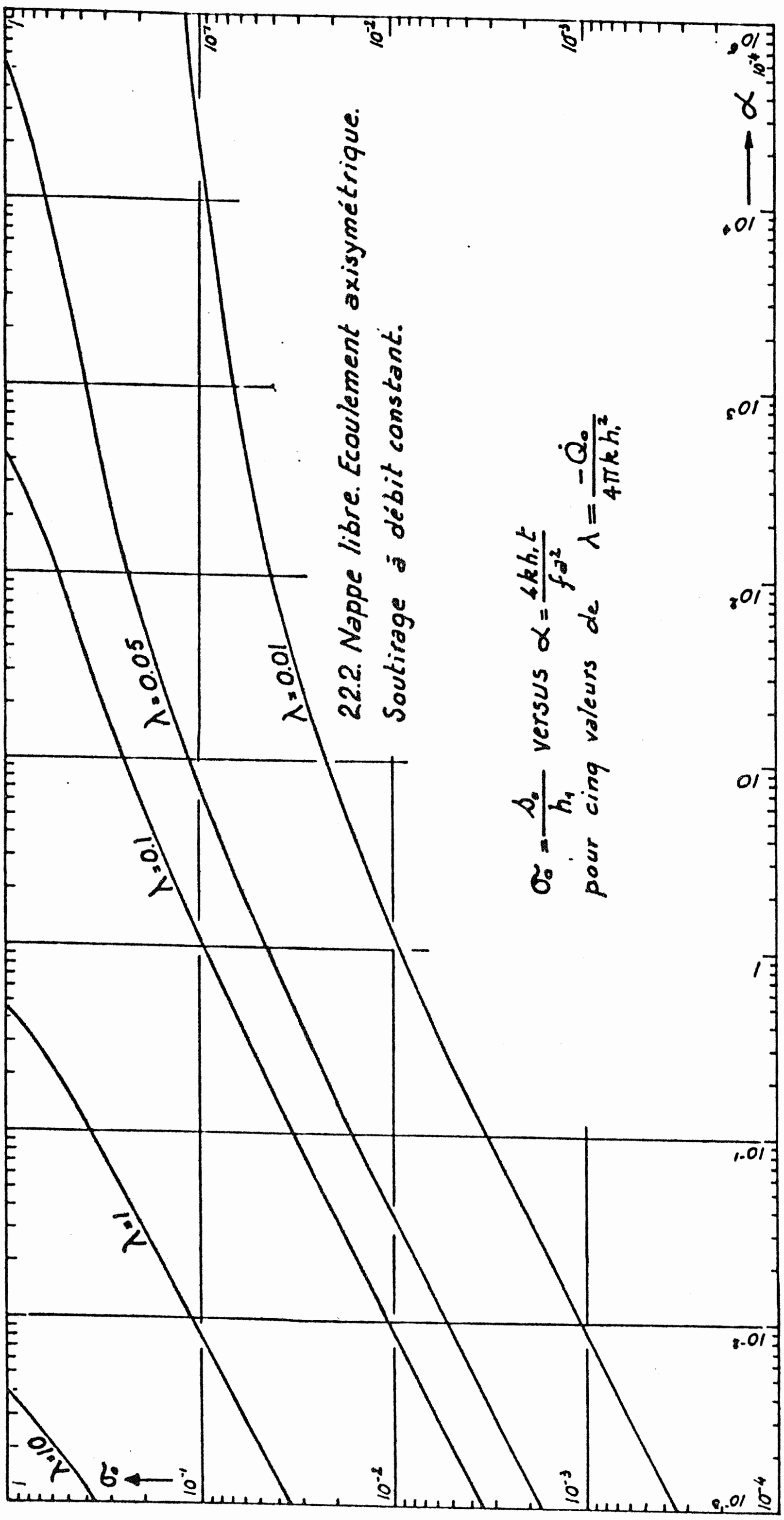
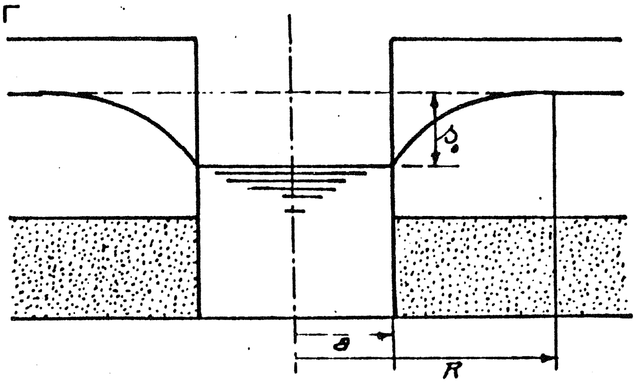

22.2. Nappe libre. Ecoulement axisymétrique.
Soutirage à débit constant.
$\sigma_0 = \frac{s_0}{h_1}$ versus $\alpha = \frac{4kh_1t}{fa^2}$
pour cinq valeurs de $\lambda = \frac{-\dot{Q}_0}{4\pi k h_1^2}$
λ=10
λ=1
λ=0.1
λ=0.05
λ=0.01
σ
α

Tableau récapitulatif. **Nappes confinées.**

		Solutions analytiques.	Solutions méthode M. Biot.
Ecoulement parallèle. $\frac{\partial^2 s}{\partial x^2} = \frac{S}{T}\frac{\partial s}{\partial t}$ $s(x,0)=0$ $s(\infty,0)=0$	Soutirage à niveau constant $s(0,t)=s_0$ $\dot{Q}_0 = T\left(\frac{\partial s}{\partial x}\right)_{x=0}$	1.11. Ingersoll, Zobel and Ingersoll, 1948 $s = s_0 \operatorname{erfc} u$ $\dot{Q}_0 = s_0\sqrt{\frac{ST}{\pi t}}$	$s = s_0\left(1-\sqrt{\frac{52}{147}}\,u\right)^2$ $\dot{Q}_0 = s_0\sqrt{\frac{ST}{\frac{156}{49}t}}$ $u^2 = \frac{Sx^2}{4Tt}$
	Soutirage à débit constant $\left(\frac{\partial s}{\partial x}\right)_{x=0} = \frac{\dot{Q}_0}{T}$	1.12. Ferris, 1950 $s = \frac{-\dot{Q}_0 x}{T}\left[\frac{e^{-u^2}}{u\sqrt{\pi}} - \operatorname{erfc} u\right]$ $s_0 = -2\dot{Q}_0\sqrt{\frac{t}{\pi ST}}$	$s = \frac{-\dot{Q}_0 x}{T}\left(\frac{1}{u}-\sqrt{\frac{82}{147}}\right)^2\frac{u}{\sqrt{\frac{392}{123}}}$ $s_0 = -2\dot{Q}_0\sqrt{\frac{t}{\frac{392}{123}TS}}$ $u^2 = \frac{Sx^2}{4Tt}$
Ecoulement axisymétrique. $\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\left(\frac{\partial s}{\partial r}\right) = \frac{S}{T}\frac{\partial s}{\partial t}$ $s(r,0)=0$ $s(\infty,0)=0$	Soutirage à niveau constant $s(a,t)=s_0$ $\dot{Q}_0 = 2\pi a T\left(\frac{\partial s}{\partial r}\right)_{r=a}$	1.21. Jacob and Lohman, 1952 $\dot{Q}_0 = -2\pi T s_0\, G(\alpha)$ $G(\alpha) = \frac{4\alpha}{\pi}\int_0^\infty x e^{-\alpha x^2}\left[\frac{\pi}{2} + tg^{-1}\frac{Y_0(x)}{J_0(x)}\right]dx$	$\dot{Q}_0 = -2\pi T s_0\,\frac{\Psi I_4}{u_0 I_1}$ $\frac{du_0}{d\alpha} = \frac{4u_0 e^{-2u_0} I_4}{I_1}$ $\alpha = \frac{Tt}{Sa^2}$
	Soutirage à débit constant $\left(r\frac{\partial s}{\partial r}\right)_{r=a} = \frac{\dot{Q}_0}{2\pi T}$	1.22. Everdingen and Hurst, 1949 $s = \frac{-\dot{Q}_0}{4\pi T}S(\tau,\rho)$ $S(\tau,\rho) = \frac{4}{\pi}\int_0^\infty \left(1-e^{-\tau u^2}\right)\frac{J_1(u)Y_0(\rho u) - Y_1(u)J_0(\rho u)}{u^2\left[J_1^2(u)+Y_1^2(u)\right]}du$ $\tau = \frac{Tt}{Sa^2}$ $\rho = \frac{r}{a}$	$s = s_0\left(1-\frac{1}{u_0}\log\rho\right)$ $s_0 = -\frac{\dot{Q}_0}{4\pi T}\,\frac{2u_0 e^{-2u_0}}{\varphi_0}\alpha$ $\frac{du_0}{d\alpha} = \frac{(I_4 e^{-2u_0}\alpha - I_2)\varphi_0}{F\alpha}$ $\alpha = \frac{4Tt}{Sa^2}$; $\rho = \frac{r}{a}$.

Tableau récapitulatif.		Nappes libres.
		solutions méthode M. Biot.
Ecoulement parallèle. $(h_1-s)\frac{\partial^2 s}{\partial x^2} - \left(\frac{\partial s}{\partial x}\right)^2 = \frac{f}{k}\frac{\partial s}{\partial t}$ $s(x,0)=0$ $s(\infty,t)=0$	Soutirage à niveau constant $\dot{Q}_0 = k(h_1-s_0)\left(\frac{\partial s}{\partial x}\right)_{x=0}$; $s(0,t)=s_0$	2.1.1 $s = s_0\left(1-\sqrt{\frac{52}{147}}\,\frac{u}{1-\frac{30}{49}\frac{s_0}{h_1}}\right)^2$ $\dot{Q}_0 = -s_0\sqrt{1-\frac{30}{49}\frac{s_0}{h_1}\,\frac{kh_1 f}{\frac{156}{49}t}}$ $u = \sqrt{\frac{fx^2}{4kh_1 t}}$
	Soutirage à débit constant $k(h_1-s_0)\left(\frac{\partial s}{\partial x}\right)_{x=0} = \dot{Q}_0$	2.1.2 $\frac{\sigma}{\lambda} = \frac{3\alpha}{\rho}\left(1-\frac{\xi}{\rho}\right)^2$ ρ donné par $\frac{d\rho}{d\alpha} = -\frac{15}{11}\frac{\rho}{\alpha} + \frac{147}{11}\frac{1}{\rho} - \frac{270\lambda}{11}\frac{\alpha}{\rho^2}$ $\sigma_0 = 3\lambda\frac{\alpha}{\rho}$ $\dot{q} = -\frac{15}{11}\frac{q}{t} + \frac{147}{11}\frac{kh_1}{fq} + \frac{270}{11}\frac{k}{f^2q^2}\dot{Q}_0 t$ $\left\{\begin{array}{l} \sigma = \frac{s}{h_1} ;\ \lambda = \frac{-\dot{Q}_0}{kh_1} ;\ \rho = \frac{q}{h_1} \\ \sigma_0 = \frac{s_0}{h_1} ;\ \alpha = \frac{kt}{fh_1} ;\ \xi = \frac{x}{h_1} \end{array}\right.$
Ecoulement axisymétrique. $\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} - \frac{1}{h_1-s}\left(\frac{\partial s}{\partial r}\right)^2 = \frac{f}{k(h_1-s)}\frac{\partial s}{\partial t}$; $s(r,0)=0$; $s(\infty,t)=0$	Soutirage à niveau constant $\dot{Q}_0 = 2\pi k a(h_1-s_0)\left(\frac{\partial s}{\partial r}\right)_{r=a}$; $s(0,t)=s_0$	2.2.1 $\frac{\dot{Q}_0}{(2\pi k h_1)s_0} = \frac{(u_0 I_4 - \sigma_0 I_7)\varphi_0}{u_0^2 I_1}$ avec $\frac{du_0}{d\alpha} = \frac{4(u_0 I_4 - \sigma_0 I_7)e^{-2u_0}}{I_1}$ $\alpha = \frac{kh_1 t}{fa^2}$ $\sigma_0 = \frac{s_0}{h_1}$
	Soutirage à débit constant. $2\pi k a(h_1-s_0)\left(\frac{\partial s}{\partial r}\right)_{r=a} = \dot{Q}_0$	2.2.2. $s = s_0\left(1-\frac{1}{u_0}\log\frac{r}{a}\right)$ $s_0 = \frac{-\dot{Q}_0}{4\pi k h_1}\,\frac{2u_0 e^{-2u_0}}{\varphi_0}\,\alpha$ $\frac{du_0}{d\alpha} = \frac{\left(I_4 e^{-2u_0}\alpha - I_2 - \frac{2e^{-4u_0}I_7\lambda}{\varphi_0}\alpha^2\right)\varphi_0}{F\alpha}$ $\alpha = \frac{4kh_1 t}{fa^2}$ $\lambda = \frac{-\dot{Q}_0}{4\pi k h_1^2}$

1.2.1. Nappe confinée. Ecoulement axisymétrique. Soutirage à débit constant : $-\dot{Q}_0$. Prise en compte de la capacité du puits.



Solution analytique. Papadopoulos and Cooper (1967)

$$\frac{s_0}{-\frac{\dot Q_0}{4\pi T}} = \frac{32 S^2}{\pi^2}\int_0^\infty \frac{1-e^{-\frac{\beta^2\alpha}{4}}}{\beta^3\,\Delta(\beta)}\,d\beta$$

avec : $\Delta(\beta) = [\beta J_0(\beta) - 2SJ_1(\beta)]^2 + [\beta Y_0(\beta) - 2SY_1(\beta)]^2$

et $\alpha = \frac{4Tt}{Sa^2}$

Dans le problème 1.2.1. la contrainte $Q_0 = \dot Q_0 t$ devient :

$Q_0 = \pi a^2 s_0 + \frac{\pi S a^2 s_0 \varphi_0}{2u_0 e^{-2u_0}}$ d'où $\dot Q_0 = \pi a^2 \dot s_0 + \frac{\pi S a^2 \varphi_0 \dot s_0}{2u_0 e^{-2u_0}} + \frac{\pi S a^2 s_0 \psi_0}{2u_0 e^{-2u_0}}\,\frac{\dot u_0}{u_0}$ et

$\frac{\dot Q_0}{Q_0} = \frac{1}{t}$ devient $\frac{1}{t} = \frac{\dot s_0}{s_0} + \frac{S\psi_0}{2u_0e^{-2u_0}+S\varphi_0}\,\frac{\dot u_0}{u_0}$ d'où

$\frac{I_1}{I_4}\frac{\dot u_0}{u_0} + \frac{I_2}{I_4}\left(\frac{1}{t} - \frac{\psi_0}{\frac{2u_0e^{-2u_0}}{S}+\varphi_0}\,\frac{\dot u_0}{u_0}\right) = \frac{4T}{Sa^2}e^{-2u_0}$ ou, en posant $\frac{4Tt}{Sa^2} = \alpha$

$\boxed{\frac{du_0}{d\alpha} = \frac{\left(I_4e^{-2u_0} - \frac{I_2}{\alpha}\right)\left(\frac{2u_0}{S} + \frac{\varphi_0}{e^{-2u_0}}\right)}{\left(\frac{2I_1}{S} + \frac{F}{e^{-2u_0}}\right)}}$ et $Q_0 = \dot Q_0 t$ devient : $\boxed{\frac{s_0}{-\frac{\dot Q_0}{4\pi T}} = \frac{2u_0e^{-2u_0}}{\frac{2u_0e^{-2u_0}}{S}+\varphi_0}\,\alpha}$

Pour u_0 petit $\frac{du_0}{d\alpha} = \frac{\left(\frac{4}{3}u_0^3\alpha - \frac{6}{5}u_0^5\right)(2u_0+25u_0^2)}{\alpha\left[\frac{64}{15}u_0^5 + \frac{28}{15}Su_0^6\right]} = \frac{(10\alpha - 9u_0^2)(1+5u_0)}{\alpha u_0(16+75u_0)} \approx \frac{10\alpha - 9u_0^2}{16\alpha u_0}$

$\frac{du_0^2}{d\alpha} = \frac{10\alpha - 9u_0^2}{8\alpha}$ d'où $u_0^2 = \frac{10}{17}\alpha$

Ci-dessous, les résultats de l'intégration de $\frac{du_0}{d\alpha} = \frac{(I_4e^{-2u_0}\alpha - I_2)(2u_0e^{-2u_0}+S\varphi_0)}{\alpha(2I_1e^{-2u_0}+SF)}$

en partant, pour $\alpha = 0.01$, de $u_0 = 0.076696499$

et du calcul de $\sigma_0 = \frac{s_0}{-\frac{\dot Q_0}{4\pi T}} = \frac{2u_0e^{-2u_0}}{\frac{2u_0e^{-2u_0}}{S}+\varphi_0}\,\alpha$ pour

diverses valeurs de S. : 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5} et 10^{-6}.

$S = 10^{-1}$

α	u_0	σ_0	α	u_0	σ_0	α	u_0	σ_0
10^{-2}	0.07669	0.0009919	10^{3}	3.47871	6.253580	10^{8}	9.22525	17.908233
1.5	0.09149	0.0014855	1.4	3.64926	6.622982	1.4	9.39302	
2	0.10432	0.0019778	2	3.82930	7.004564	2	9.57123	
3	0.12592	0.0029594	3	4.03249	7.438396	3	9.77350	
4	0.14398	0.0039374	4	4.17704	7.732287	4	9.91725	19.295835
5	0.15970	0.0049124	5	4.28879	7.960661	5	10.02868	
6	0.17375	0.0058848	6	4.37938	8.155226	6	10.11913	
7	0.18652	0.0068546	7	4.45650	8.310885	7	10.19639	
8	0.19829	0.0078221	8	4.52329	8.444901	8	10.26322	
9	0.20923	0.0087875	9	4.58215	8.563390	9	10.32209	
10^{-1}	0.21948	0.0097509	10^{4}	4.63475	8.669644	10^{9}	10.37473	20.212968
1.5	0.26330	0.0145401	1.4	4.80192	9.019507			
2	0.29894	0.0192889	2	4.98005	9.372540			
3	0.35625	0.0286851	3	5.17966	9.821667			
4	0.40234	0.0379651	4	5.32463	10.08049			
5	0.44139	0.0471437	5	5.43615	10.297033			
6	0.47553	0.0562313	6	5.52703	10.478045			
7	0.50601	0.0652354	7	5.60189	10.669112			
8	0.53363	0.0741619	8	5.66939	10.783553			
9	0.55895	0.0830156	9	5.72848	10.892990			
1	0.58236	0.0918004	10^{5}	5.78114	10.994692			
1.5	0.67936	0.134796	1.4	5.94714	11.363989			
2	0.75492	0.176439	2	6.12621	11.691384			
3	0.87097	0.256344	3	6.31985	12.284179			
4	0.96008	0.332443	4	6.46945	12.432069			
5	1.03302	0.405297	5	6.58243	12.614233			
6	1.09507	0.475295	6	6.66982	12.878822			
7	1.14922	0.542731	7	6.74686	13.028901			
8	1.19737	0.607839	8	6.81494	13.126124			
9	1.2407	0.670812	9	6.87471	13.218150			
10	1.2803	0.731810	10^{6}	6.92774	13.310863			
1.5	1.43905	1.011435	1.4	7.09590				
2	1.55762	1.256747	2	7.27377				
3	1.73280	1.671876	3	7.47586				
4	1.86252	2.013572	4	7.61936	14.685863			
5	1.96605	2.302079	5	7.73065				
6	2.05240	2.550236	6	7.82136				
7	2.12657	2.766797	7	7.89834				
8	2.19161	2.958020	8	7.96500				
9	2.24954	3.128546	9	8.02377				
10^{2}	2.30178	3.281896	10^{7}	8.07634	15.603253			
1.5	2.50586	3.869674	1.4	8.24407				
2	2.65280	4.275025	2	8.42212				
3	2.86141	4.818935	3	8.62434				
4	3.00962	5.184080	4	8.76797	16.991035			
5	3.124414	5.456104	5	8.87934				
6	3.21797	5.672088	6	8.96998				
7	3.29689	5.850871	7	9.04709				
8	3.36509	6.003241	8	9.11382				
9	3.42511	6.136080	9	9.17265				
10^{3}	3.47871	6.253580	10^{8}	9.22525	17.908233			

$S = 10^{-2}$					
α	U_0	σ_0	α	U_0	σ_0
10^{-2}	0.07669	0.00009991	10^{3}	3.34823	4.550795
1.5	0.09144	0.00014726	1.4	3.53046	5.304523
2	0.10424	0.00019977	2	3.72640	6.055440
3	0.12578	0.00029959	3	3.95032	6.810557
4	0.14378	0.00039936	4	4.10863	7.276977
5	0.15945	0.00049911	5	4.230440	7.604812
6	0.17345	0.0005988	6	4.32913	8.150600
7	0.18617	0.00069985	7	4.40098	8.195885
8	0.19789	0.0007982	8	4.47787	8.294372
9	0.20879	0.0008978	9	4.54295	8.405490
10^{-1}	0.21899	0.0009974	10^{4}	4.59984	8.515153
1.4	0.25466	0.0013957	1.4	4.77694	8.890728
2	0.29801	0.0019926	2	4.96160	9.282851
3	0.35491	0.0029863	3	5.16805	9.736300
4	0.40061	0.0039787	4	5.31486	10.028646
5	0.43928	0.0049700	5	5.42797	10.257931
6	0.47306	0.0059603	6	5.51896	10.465651
7	0.50318	0.0069496	7	5.59690	10.619377
8	0.53046	0.0079380	8	5.66439	10.750643
9	0.55544	0.0089255	9	5.72376	10.867630
1	0.57853	0.0099123	10^{5}	5.77677	10.973227
1.5	0.67398	0.0148347	1.4	5.94435	11.338940
2	0.74810	0.0197402	2	6.12388	11.679651
3	0.86154	0.0295070	3	6.32019	12.231491
4	0.94826	0.0392211	4	6.46875	12.413398
5	1.01899	0.0488876	5	6.58146	12.608019
6	1.07895	0.058510	6	6.669311	12.866186
7	1.13113	0.068091	7	6.74654	13.014750
8	1.17740	0.077632	8	6.81453	13.116637
9	1.21901	0.087136	9	6.87422	13.21266
10	1.25686	0.096603	10^{6}	6.92724	13.307338
1.5	1.40776	0.1434351	4	7.619210	14.685198
2	1.51957	0.189494	10^{7}	8.07628	15.602970
3	1.68333	0.279560	4	8.76795	16.990957
4	1.80356	0.367169	10^{8}	9.225247	17.908200
5	1.89899	0.452559	4	9.91725	19.295826
6	1.97830	0.535903	10^{9}	10.37472992	20.21296462
7	2.04628	0.617337			
8	2.10582	0.696974			
9	2.15885	0.774909			
10^{2}	2.20688	0.851227			
1.4	2.36194	1.141687			
2	2.53066	1.53869			
3	2.72778	2.11685			
4	2.87120	2.612227			
5	2.98454	3.042079			
6	3.07851	3.418770			
7	3.15893	3.731529			
8	3.22928	4.047523			
9	3.29186	4.312424			
10^{3}	3.34823	4.550795			

3.

$S=10^{-3}$					
α	u_0	σ_0	α	u_0	σ_0
10^{-2}	0.07669	0.00000999	10^{3}	3.26080	0.906564
1.5	0.09143	0.00001499	1.4	3.42509	1.231668
2	0.10423	0.00001999	2	3.60166	1.68715
3	0.12576	0.00002999	3	3.80579	2.37257
4	0.14376	0.00003999	4	3.95320	2.98014
5	0.15943	0.00004999	5	4.06919	3.52231
6	0.17342	0.00005998	6	4.16512	4.00866
7	0.18614	0.00006998	7	4.24711	4.44685
8	0.19785	0.000073982	8	4.31879	4.84315
9	0.20874	0.000089978	9	4.38256	5.20282
10^{-1}	0.21894	0.000099975	10^{4}	4.44001	5.53026
1.4	0.25460	0.00013995	1.4	4.62609	6.58544
2	0.29792	0.00019992	2	4.82714	7.65237
3	035477	0.00029986	3	5.05848	8.70517
4	0.40043	0.00039978	4	5.22275	9.32143
5	0.43906	0.00049969	5	5.34924	9.73019
6	0.47280	0.00059960	6	5.45150	10.02869
7	0.50289	0.00069949	7	5.53720	10.25932
8	0.53013	0.00079937	8	5.61078	10.44659
9	0.55507	0.00089925	9	5.67517	10.60414
1	0.57813	0.000999117	10^{5}	5.73236	10.74013
1.5	0.67341	0.0014983	1.4	5.91233	11.15600
2	0.74737	0.0019973	2	6.10009	11.56332
3	0.86052	0.0029950	3	6.30881	12.03834
4	0.94696	0.0039920	4	6.45726	12.32692
5	1.01743	0.0049886	5	6.57130	12.55610
6	1.07714	0.0059848	6	6.66235	12.78268
7	1.12308	0.0069805	7	6.74082	12.93428
8	1.17511	0.0079757	8	6.80886	13.06012
9	1.21650	0.00897065	9	6.86865	13.17372
10	1.25412	0.00996513	10^{6}	6.92196	13.27765
1.5	1.40397	0.014932	4	7.61768	14.67854
2	1.51481	0.0198906	10^{7}	8.07562	15.60013
3	1.67675	0.0297846	4	8.76778	16.99018
4	1.79530	0.0396503	10^{8}	9.22517	17.90787
5	1.88914	0.0494896	4	9.91723	19.29573
6	1.96694	0.0593039	10^{9}	10.37472	20.21292
7	2.03345	0.0690943			
8	2.09159	0.0788615			
9	2.14325	0.0886062			
10^{2}	2.18976	0.0983291			
1.4	2.33999	0.1370112			
2	2.50188	0.1944442			
3	2.68888	0.2887170			
4	2.82332	0.3813149			
5	2.92857	0.4723493			
6	3.01518	0.561505			
7	3.08883	0.650050			
8	3.15296	0.736842			
9	3.20977	0.82233			
10^{3}	3.26080	0.9065648			

$S = 10^{-4}$

α	U_0	σ_0	α	U_0	σ_0
10^{-2}	0.07669	10^{-6}	10^{3}	3.24873	0.0990005
1.5	0.09143	$1.5 \cdot 10^{-6}$	1.4	3.40915	0.138162
2	0.10423	$2 \cdot 10^{-6}$	2	3.58029	0.196489
3	0.12576	$3 \cdot 10^{-6}$	3	3.77611	0.292651
4	0.14376	$4 \cdot 10^{-6}$	4	3.91587	0.38757
5	0.15942	$5 \cdot 10^{-6}$	5	4.02474	0.481324
6	0.17342	$6 \cdot 10^{-6}$	6	4.11402	0.573944
7	0.18614	$7 \cdot 10^{-6}$	7	4.18973	0.665474
8	0.19785	$8 \cdot 10^{-6}$	8	4.25550	0.755948
9	0.20874	$9 \cdot 10^{-6}$	9	4.31366	0.845394
10^{-1}	0.21894	10^{-5}	10^{4}	4.36581	0.933840
1.4	0.25459	$1.4 \cdot 10^{-5}$	1.4	4.53321	1.278070
2	0.29791	$1.9999 \; 10^{-5}$	2	4.71229	1.767805
3	0.35476	$2.9999 \; 10^{-5}$	3	4.91839	2.520821
4	0.40041	$3.9998 \; 10^{-5}$	4	5.06664	3.204484
5	0.43904	$4.9997 \; 10^{-5}$	5	5.18302	3.827595
6	0.47277	$5.9996 \; 10^{-5}$	6	5.27913	4.397273
7	0.50286	$6.9995 \; 10^{-5}$	7	5.36118	4.919436
8	0.53009	$7.9994 \; 10^{-5}$	8	5.43289	5.399136
9	0.55504	$8.9993 \; 10^{-5}$	9	5.49665	5.840726
1	0.57809	$9.9991 \; 10^{-5}$	10^{5}	5.55411	6.247999
1.5	0.67335	$1.49983 \; 10^{-4}$	1.4	5.74038	7.594892
2	0.74730	$1.99974 \; 10^{-4}$	2	5.94237	9.007937
3	0.86041	$2.99950 \; 10^{-4}$	3	6.17648	10.434349
4	0.94683	$3.99921 \; 10^{-4}$	4	6.34397	11.26094
5	1.01727	$4.99887 \; 10^{-4}$	5	6.47354	11.789968
6	1.07696	$5.99848 \; 10^{-4}$	6	6.57854	12.159584
7	1.12887	$6.99805 \; 10^{-4}$	7	6.66654	12.434175
8	1.17488	$7.99757 \; 10^{-4}$	8	6.74207	12.649796
9	1.21624	$8.99706 \; 10^{-4}$	9	6.80808	12.826416
10	1.25385	$9.99650 \; 10^{-4}$	10^{6}	6.86664	12.97578
1.5	1.40358	0,00149931	4	7.60226	14.609408
2	1.51432	0.001998902	10^{7}	8.06907	15.571417
3	1.67607	0.0029978	4	8.76599	16.982397
4	1.79444	0.0039964	10^{8}	9.22441	17.904573
5	1.88811	0.0049948	4	9.91702	19.294836
6	1.96575	0.0059929	10^{9}	10.37463	20.2125444
7	2.03210	0.0069908			
8	2.09009	0.0079884			
9	2.14160	0.0089859			
10^{2}	2.18795	0.00998308			
1.5	2.36859	0.014965			
2	2.49869	0.019943			
3	2.68443	0.029884			
4	2.81768	0.039806			
5	2.92178	0.049712			
6	3.00727	0.0596013			
7	3.07985	0.0694742			
8	3.14292	0.0793315			
9	3.19871	0.0891735			
10^{3}	3.24873	0.0990005			

$S = 10^{-5}$								
α	u_0	σ_0	α	u_0	σ_0	α	u_0	σ_0
10^{-2}	0.07669	10^{-7}	10^{3}	3.24748	$9.989936 . 10^{-3}$	10^{8}	9.21681	17.87128
1.5	0.09143	1.5 . ″	1.4	3.40749		4	9.91498	1928585
2	0.10423	2 . ″	2	3.57804		10^{9}	10.37377822	20.2087358
3	0.12576	3 . ″	3	3.77291				
4	0.14376	4 . ″	4	3.91176	0.039873			
5	0.15942	5 . ″	5	4.01976	0.049808			
6	0.17342	6 . ″	6	4.10818				
7	0.18614	7 . ″	7	4.18307				
8	0.19785	8 . ″	8	4.24802				
9	0.20874	9 . ″	9	4.30537				
10^{-1}	0.21894	10^{-6}	10^{4}	4.35673	0.0993 0767			
1.5	0.26251	1.5 . ″	1.4	4.52106				
2	0.29791	2 ″	2	4.69576				
3	0.35476	3 ″	3	4.89502				
4	0.40041	4 ″	4	5.03684	0.390805			
5	0.43904	5 ″	5	5.14713	0.486047			
6	0.47277	6 ″	6	5.23745				
7	0.50285	7 ″	7	5.31396				
8	0.53009	8 ″	8	5.38036				
9	0.55503	9 ″	9	5.43904				
1	0.57808	10^{-5}	10^{5}	5.49161	0.949130			
1.5	0.67335	1.5 ″	1.4	5.66012				
2	0.74729	2 ″	2	5.84001				
3	0.86041	3 ″	3	6.04653				
4	0.94682	$3.9999\ 10^{-5}$	4	6.19475	3.350318			
5	1.01726	4.9999 ″	5	6.31091	4.031835			
6	1.07694		6	6.40672				
7	1.12885		7	6.48844				
8	1.17486		8	6.55982				
9	1.21622		9	6.62325				
10	1.25382	$9.9997\ 10^{-5}$	10^{6}	6.68041	6.779693			
1.5	1.40354		1.4	6.86568				
2	1.51427	$1.99989\ 10^{-4}$	2	7.06694				
3	1.67600		3	7.30138				
4	1.79436	$3.99965\ 10^{-4}$	4	7.47023	13.06517			
5	1.88801	$4.99949\ 10^{-4}$	5	7.60159				
6	1.96563		6	7.70853				
7	2.03197		7	7.79830				
8	2.08994		8	7.87540				
9	2.14143		9	7.94281				
10^{2}	2.18777	$9.99831 . 10^{-4}$	10^{7}	8.00258	15.19584			
1.5	2.36833		1.4	8.19000				
2	2.49837	$1.999432 . 10^{-3}$	2	8.38339				
3	2.68398		3	8.59803				
4	2.81710	$3.998061 . 10^{-3}$	4	8.74788	16.90147			
5	2.92108	$4.997112 . 10^{-3}$	5	8.86307				
6	3.00647		6	8.95654				
7	3.07893		7	9.03536				
8	3.14190		8	9.10343				
9	3.19757		9	9.16333				
10^{3}	3.24748	$9.989936 . 10^{-3}$	10^{8}	9.21681	17.87128			

$$S = 10^{-6}$$

α	u_0	σ_0
Pour $\alpha \lesssim 100$, $\sigma_0 = 10^{-6} \times \alpha$		
10^2	2.187756304	$9.9998 \cdot 10^{-5}$
4	2.817047689	$3.9998 \cdot 10^{-4}$
10^3	3.247362810	$9.9989 \cdot 10^{-4}$
4	3.911354370	$3.9987 \cdot 10^{-3}$
10^4	4.3558	0.0099930
4	5.0336	0.0399066
10^5	5.4844	0.0994737
4	6.1702	0.392720
10^6	6.6270	0.958740
4	7.3305	3.451633
10^7	7.8139	7.184462
4	8.6004	14.723249
10^8	9.1395	17.396259
4	9.8935	19.214134
10^9	10.36459	20.190758

Le rabattement s à une distance r de l'axe du puits est donné par :

$$\sigma = \frac{s}{\left(\frac{-\dot{Q}_0}{4\pi T}\right)} = \sigma_0 \left(1 - \frac{1}{u_0} \log \frac{r}{a}\right)$$

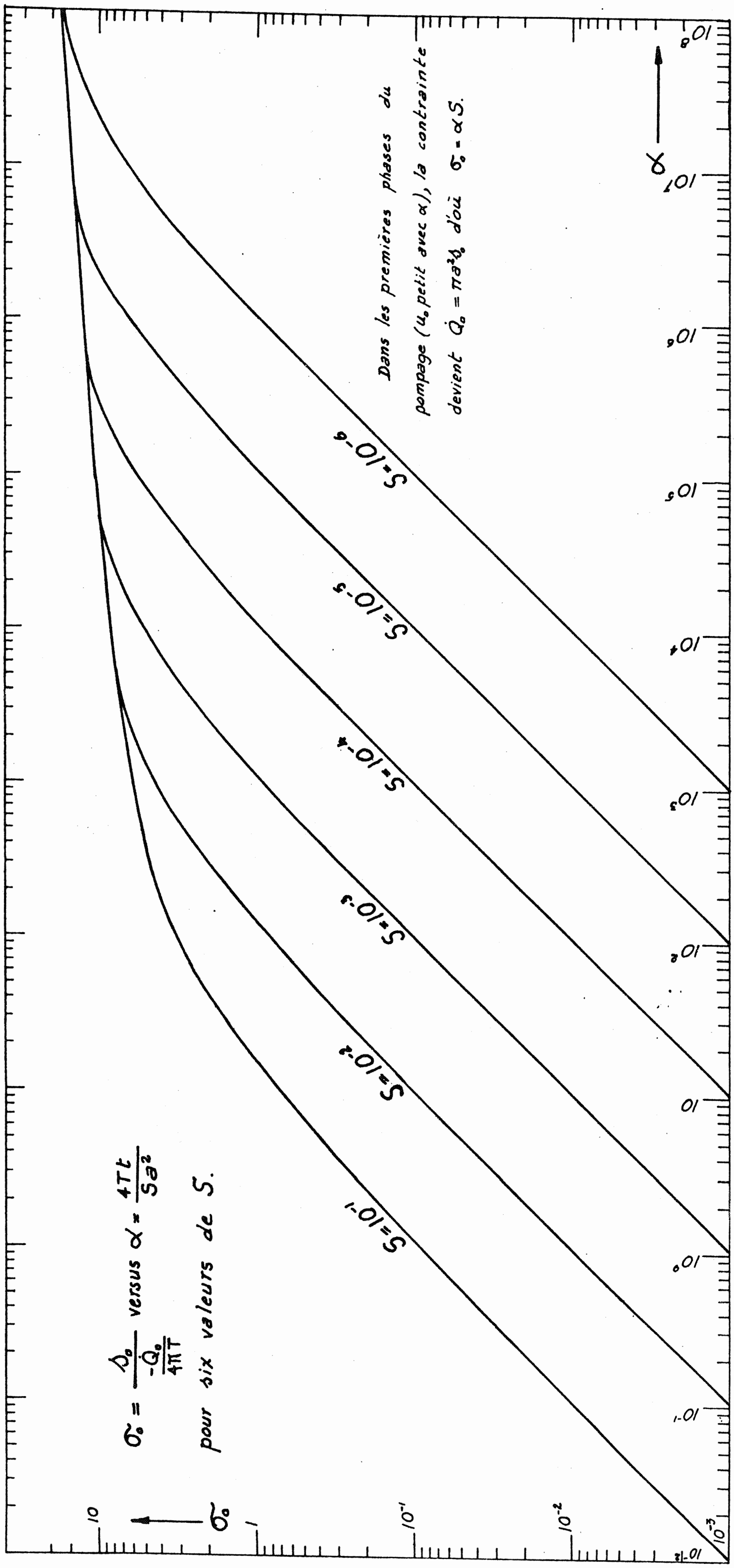
α	$S=10^{-1}$		$S=10^{-2}$		$S=10^{-3}$		$S=10^{-4}$		$S=10^{-5}$	
	1	2	1	2	1	2	1	2	1	2
0.1	$9.755.10^{-3}$	$9.751.10^{-3}$	$9.976.10^{-4}$	$9.974.10^{-4}$	$9.998.10^{-5}$	9999.10^{-5}	1.10^{-5}	1.10^{-5}	1.10^{-6}	1.10^{-6}
1	$9.192.10^{-2}$	$9.180.10^{-2}$	$9.914.10^{-3}$	$9.912.10^{-3}$	$9.991.10^{-4}$	$9.991.10^{-4}$	1.10^{-4}	1.10^{-4}	1.10^{-5}	1.10^{-5}
2	$1.767.10^{-1}$	$1.764.10^{-1}$	$1.974.10^{-2}$	$1.974.10^{-2}$	$1.997.10^{-3}$	$1.997.10^{-3}$	2.10^{-4}	2.10^{-4}	2.10^{-5}	2.10^{-5}
5	4.062 "	4.053 "	4.890 "	4.889. "	4.988 "	4.989 "	4.999 "	4.999 "	5. "	4.9999 "
10	7.336 "	7.318 "	9.665 "	9.660 "	9.966 "	9.965 "	9.997 "	9.996 "	1.10^{-4}	9.9997 "
20	1.260	1.257	$1.896.10^{-1}$	$1.895.10^{-1}$	$1.989.10^{-2}$	$1.989.10^{-2}$	$1.999.10^{-3}$	$1.999.10^{-3}$	2. "	$1.99989\,10^{-4}$
50	2.303	2.302	4.529. "	4.525 "	4.949 "	4.949 "	4.995 "	4.995 "	5. "	4.99949 "
100	3.276	3.282	8.520 "	8.512 "	9.834 "	9.833 "	9.984 "	9.983 "	1.10^{-3}	9.998311 "
200	4.255	4.275	1.540	1.539	$1.945.10^{-1}$	$1.944.10^{-1}$	$1.994.10^{-2}$	$1.994.10^{-2}$	2. "	$1.999431\,10^{-3}$
500	5.420	5.456	3.043	3.042	4.725 "	4.723 "	4.972 "	4.971 "	4.998 "	4.9971 "
10^3	6.212	6.253	4.545	4.551	9.069 "	9.066 "	9.901 "	9.900 "	9.992 "	9.9899 "
2.10^3	6.960	7.004	6.031	6.055	1.688	1.687	$1.965.10^{-1}$	$1.965.10^{-1}$	$1.997.10^{-2}$	$1.9964.10^{-2}$
5. "	7.866	7.961	7.557	7.605	3.523	3.522	4.814 "	4.813 "	4.982 "	4.9808 "
10^4	8.572	8.669	8.443	8.515	5.526	5.530	9.340 "	9.338 "	9.932 "	9.9307 "
2.10^4	9.318	9.372	9.229	9.283	7.631	7.652	1.768 "	1.760	$1.975.10^{-1}$	$1.9748.10^{-1}$
5 "	10.24	10.29	10.20	10.26	9.676	9.730	3.828	3.827	4.861 "	4.86047 "
10^5	10.93	10.99	10.87	10.97	10.68	10.74	6.245	6.248	9.493 "	9.4913 "
2 "	11.63	11.69	11.62	11.68	11.50	11.56	8.991	9.008	1.817	1.81623
5 "	12.55	12.61	12.54	12.61	12.49	12.55	11.74	11.79	4.033	4.0318
10^6	13.24	13.31	13.24	13.31	13.21	13.28	12.91	12.97	6.779	6.7796

$$\frac{s_o}{\left(\frac{-\dot{Q}_o}{4\pi T}\right)} \text{ versus } \alpha = \frac{4Tt}{Sa^2} \text{ pour cinq valeurs de } S.$$

Comparaison entre les valeurs obtenues par la formule de Papadopoulos and Cooper (1967) 1 et celles que fournit la présente méthode 2.

$\frac{R}{a}$ versus $\alpha = \frac{4Tt}{Sa^2}$ pour cinq valeurs de S.

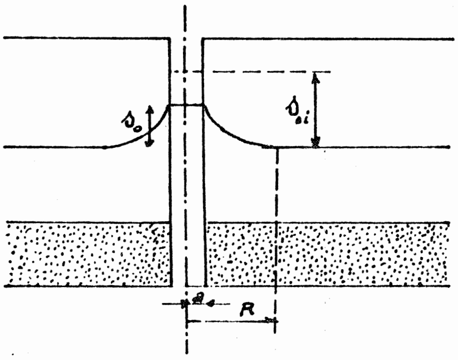
α	$S=10^{-1}$	$S=10^{-2}$	$S=10^{-3}$	$S=10^{-4}$	$S=10^{-5}$
1	1.790	1.783	1.782	1.782	1.782
10	3.597	3.514	3.504	3.503	3.503
10^2	9.991	9.085	8.933	8.916	8.915
10^3	32.41	28.45	26.07	25.75	25.72
10^4	103.0	99.46	84.77	78.71	78.00
10^5	324.1	322.7	308.6	258.2	242.6
10^6	1020.	1019.	1014.	959.7	796.6
10^7	3217.	3217.	3215.	3194.	2988.
10^8	10150.	10150	10149.	10141.	10064.


Dans les premières phases du pompage (u_0 petit avec α), la contrainte devient $\dot{Q}_0 = \pi a^2 \dot{s}_0$ d'où $\sigma_0 = \alpha S$.
$\sigma_0 = \dfrac{s_0}{\frac{-Q_0}{4\pi T}}$ versus $\alpha = \dfrac{4Tt}{Sa^2}$
pour six valeurs de S.
$S=10^{-1}$
$S=10^{-2}$
$S=10^{-3}$
$S=10^{-4}$
$S=10^{-5}$
$S=10^{-6}$
α
σ_0
10^{-2}
10^{-1}
10^{0}
10
10^{2}
10^{3}
10^{4}
10^{5}
10^{6}
10^{7}
10^{8}
10
1
10^{-1}
10^{-2}
10^{-3}

4. Essai par choc hydraulique (Slug test).

Nappe confinée. Ecoulement axisymétrique.

Soit une injection instantanée d'un volume d'eau V dans un puits de diamètre a. Le niveau de l'eau dans le puits s'élève de $s_{oi} = \frac{V}{\pi a^2}$ puis retourne à son niveau initial suivant une certaine fonction du temps qui fait l'objet du problème.



Solution analytique. Cooper, Bredehoeft and Papadopulos (1967)

$$\frac{s_o}{s_{oi}} = \frac{8S}{\pi^2} \int_0^\infty \frac{e^{-\beta \frac{u^2}{S}} \, du}{u\left\{\left[u J_0(u) - 2S J_1(u)\right]^2 + \left[u Y_0(u) - 2S Y_1(u)\right]^2\right\}}$$

$$\beta = \frac{Tt}{a^2}$$

Les deux coordonnées généralisées sont ici s_o et u_o, dans ce cas, on a (problème 1.2.2) :

$\frac{I_1}{I_4}\frac{\dot{u}_o}{u_o} + \frac{I_2}{I_4}\frac{\dot{s}_o}{s_o} = \frac{4T}{Sa^2} e^{-2u_o}$. La contrainte du problème exprime que la diminution du volume d'eau dans le puits est égale à l'augmentation du volume d'eau dans le terrain, soit :

$$\pi a^2 \dot{s}_o = 2\pi a T \left(\frac{\partial s}{\partial r}\right)_{r=a}$$

or si $s = s_o \frac{u}{u_o}$, $\frac{\partial s}{\partial r} = -\frac{1}{r}\frac{s_o}{u_o}$ et $\frac{\dot{s}_o}{s_o} = \frac{-2T}{a^2 u_o}$ d'où :

$$\frac{I_1}{I_4}\frac{\dot{u}_o}{u_o} - \frac{2T}{a^2 u_o}\frac{I_2}{I_4} = \frac{4T}{Sa^2} e^{-2u_o}$$ ou, en posant $\beta = \frac{Tt}{a^2}$

$$\frac{du_o}{d\beta} = \frac{2I_2 + 4u_o e^{-2u_o} I_4/S}{I_1}$$ Pour u_o petit $\frac{du_o}{d\beta} = 2\frac{\frac{6}{5}u_o^5}{\frac{32}{15}u_o^5} + \frac{\frac{4.4}{3}u_o^4\frac{1}{S}}{\frac{32}{15}u_o^5} = \frac{5}{2Su_o} + \frac{9}{8}$

dont la solution : $\frac{8}{9}u_o - \frac{160}{81S}\log\left(1 + \frac{9S}{20}u_o\right) = \beta$ peut se mettre sous la forme :

$$\frac{Su_o^2}{5} - \frac{3}{50}S^2u_o^3 - \beta = 0 \quad \text{ou} \quad u_o^3 - \frac{10}{3S}u_o^2 + \frac{50}{3S^2}\beta = 0$$

Pour	$S = 0.25$	et $\beta = 10^{-3}$	$u_o = 0.142181472$
	$S = 0.1$	et $\beta = 10^{-3}$	$u_o = 0.224363160$
	$S = 0.01$	et $\beta = 10^{-4}$	$u_o = 0.223681900$
	$S = 10^{-3}$	et $\beta = 10^{-5}$	$u_o = 0.223614$
	$S = 10^{-4}$	et $\beta = 10^{-6}$	$u_o = 0.22361$
	$S = 10^{-5}$	et $\beta = 10^{-7}$	$u_o = 0.2236$
	$S = 10^{-6}$	et $\beta = 10^{-8}$	$u_o = 0.224$
	$S = 10^{-10}$	et $\beta = 10^{-12}$	$u_o = 0.224$

$Tt/a^2 = \beta$	$S=0.25$ u_0	δ_0/δ_{0i}	$S=0.1$ u_0	δ_0/δ_{0i}
10^{-3}	0.14218	0.962307	0.22436	0.974464
1.5	0.16845	0.954863	0.26113	0.969631
2	0.19049	0.948501	0.29192	0.965427
3	0.22696	0.937736	0.34260	0.958191
4	0.25709	0.928620	0.38411	0.951957
5	0.28313	0.920579	0.41966	0.946386
6	0.30625	0.913312	0.45099	0.941298
7	0.32716	0.906636	0.47912	0.936580
8	0.34633	0.900432	0.50472	0.932160
9	0.36408	0.894615	0.52829	0.927985
10^{-2}	0.38064	0.889124	0.55015	0.924017
1.5	0.45071	0.865220	0.64136	0.906457
2	0.50689	0.845285	0.71303	0.891465
3	0.59586	0.812365	0.82408	0.866047
4	0.66634	0.785172	0.91008	0.844452
5	0.72543	0.761669	0.98097	0.825365
6	0.77668	0.740804	1.04161	0.808099
7	0.82216	0.721946	1.09481	0.792238
8	0.86319	0.704684	1.14233	0.777509
9	0.90067	0.688729	1.18537	0.763719
10^{-1}	0.93524	0.673871	1.22477	0.750725
1.5	1.07787	0.611488	1.38452	0.694607
2	1.18890	0.562309	1.50605	0.648615
3	1.36073	0.486694	1.68994	0.574943
4	1.49491	0.429393	1.83046	0.516744
5	1.60699	0.383529	1.94605	0.468686
6	1.70441	0.345578	2.04533	0.427913
7	1.79134	0.313458	2.13307	0.392680
8	1.87035	0.285823	2.21219	0.361819
9	1.94317	0.261745	2.28463	0.334509
1	2.01098	0.240558	2.35170	0.310142
1.5	2.29948	0.163987	2.63353	0.219232
2	2.53647	0.116712	2.86178	0.160546
3	2.93055	0.063760	3.23741	0.091727
4	3.26461	0.037126	3.55404	0.055325
5	3.56239	0.022569	3.83607	0.034625
6	3.83493	0.014171	4.09458	0.022286
7	4.08841	0.0091334	4.33558	0.014673
8	4.32667	0.0060152	4.56275	0.0098465
9	4.55237	0.0040357	4.77853	0.0067164
10	4.76739	0.0027516	4.98467	0.0046476
15	5.72361	0.00048831	5.90749	0.00087279
20	6.54295	0.00010856	6.70485	0.00020117

β	$S=10^{-2}$ u_0	$S=10^{-2}$ s_0/s_{0i}	$S=10^{-3}$ u_0	$S=10^{-3}$ s_0/s_{0i}	$S=10^{-4}$ u_0	$S=10^{-4}$ s_0/s_{0i}
10^{-4}	0.22368	0.997395				
↓	↓	↓	↓	↓	↓	↓
10^{-3}	0.54523	0.991947	1.18757	0.996903	2.09999	0.998538
1.5	0.63440	0.989952	1.33265		2.27824	
2	0.70415	0.988206	1.44038	0.995181	2.40685	0.997567
3	0.81159	0.985154	1.59833		2.59074	
4	0.89421	0.982467	1.71434		2.72284	
5	0.96186	0.980016	1.80636	0.991096	2.82614	0.995100
6	1.01939	0.977736	1.88277		2.91104	
7	1.06957	0.975586	1.94819		2.98314	
8	1.11417	0.973541	2.00543		3.04583	
9	1.15434	0.971582	2.05633		3.10130	
10^{-2}	1.19094	0.969697	2.10219	0.985519	3.15104	0.991529
1.5	1.33733	0.961074	2.28131	0.980598	3.34356	0.988261
2	1.44630	0.953371	2.41075	0.976045	3.48113	0.985169
3	1.60657	0.939630	2.59621	0.967628	3.67636	0.979327
4	1.72475	0.927321	2.72981	0.959820	3.81582	0.973793
5	1.81885	0.915981	2.83455	0.952436	3.92454	0.968480
6	1.89728	0.905362	2.92086	0.945377	4.01375	0.963337
7	1.96465	0.895317	2.99434	0.938579	4.08946	0.958335
8	2.02379	0.885743	3.05837	0.932001	4.15527	0.953452
9	2.07657	0.876569	3.11517	0.925613	4.21349	0.948673
10^{-1}	2.12427	0.867742	3.16622	0.919391	4.26573	0.943986
1.5	2.31221	0.827620	3.36508	0.890266	4.46827	0.921669
2	2.45004	0.792370	3.50875	0.863639	4.61372	0.900794
3	2.65148	0.731519	3.71573	0.815629	4.82192	0.862179
4	2.80038	0.679614	3.86654	0.772767	4.97258	0.826751
5	2.91998	0.634142	3.98636	0.733830	5.09162	0.793843
6	3.02081	0.593656	4.08649	0.698085	5.19061	0.763047
7	3.10857	0.557207	4.17297	0.665033	5.27574	0.734078
8	3.18670	0.524122	4.24943	0.634309	5.35071	0.706728
9	3.25742	0.493901	4.31823	0.605633	5.41791	0.680831
1	3.32227	0.466156	4.38097	0.578778	5.47899	0.656252
1.5	3.58873	0.355468	4.63528	0.466299	5.72436	0.549739
2	3.79857	0.276901	4.83182	0.380636	5.91150	0.464429
3	4.13565	0.174946	5.14148	0.260315	6.20193	0.337276
4	4.41483	0.114563	5.39344	0.182341	6.43459	0.248395
5	4.66177	0.076922	5.61394	0.129930	6.63596	0.186045
6	4.88781	0.052652	5.81457	0.093827	6.81777	0.140335
7	5.09895	0.036610	6.00144	0.068503	6.98621	0.106691
8	5.29870	0.025797	6.17811	0.050485	7.14493	0.081662
9	5.48933	0.018391	6.34685	0.037514	7.29625	0.062876
10	5.67235	0.013246	6.50916	0.0280807	7.44169	0.048670
15	6.50379	0.0029095	7.25279	0.00722259	8.10960	0.014447
20	7.23722	0.00074835	7.91932	0.002090116	8.71386	0.00468204

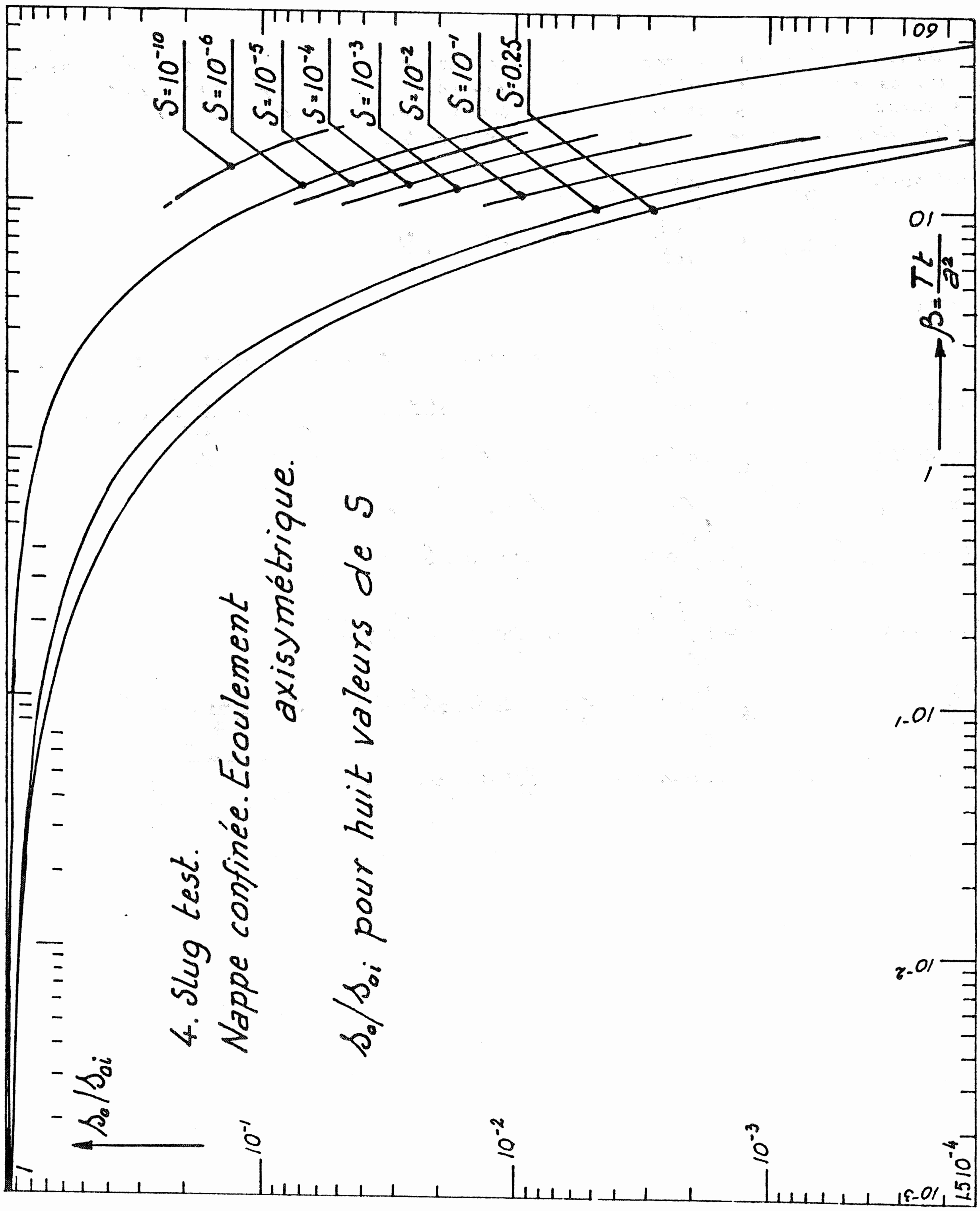
β	$S=10^{-5}$ u_0	$S=10^{-5}$ s_0/s_{0i}	$S=10^{-6}$ u_0	$S=10^{-6}$ s_0/s_{0i}	$S=10^{-10}$ u_0	$S=10^{-10}$ s_0/s_{0i}
10^{-7}	0.2236	0.9999974				
↓	↓	↓	↓	↓	↓	↓
10^{-3}	3.14953	0.999148	4.25336	0.999419	8.79865	0.999750
1.5	3.34141		4.45054		9.00028	
2	3.47839	0.998504	4.59079	0.998944	9.14338	0.999522
3	3.67246		4.78889		9.34512	
4	3.81080		4.92974		9.48830	
5	3.91844	0.996790	5.03915	0.997643	9.59939	0.998867
6	4.00659		5.12864		9.69018	
7	4.08126		5.20436		9.76695	
8	4.14604		5.27001		9.83346	
9	4.20325		5.32795		9.89214	
10^{-2}	4.25448	0.994216	5.37982	0.995644	9.94464	0.997821
1.5	4.45214	0.991809	5.57967	0.993748	10.14674	0.996804
2	4.59286	0.989500	5.72174	0.991914	10.29023	0.995805
3	4.79188	0.985079	5.922415	0.988372	10.49262	0.993845
4	4.93360	0.980838	6.06513	0.984947	10.63638	0.991922
5	5.04387	0.976728	6.17607	0.981607	10.74799	0.990025
6	5.13420	0.972721	6.26689	0.978336	10.83927	0.988151
7	5.21076	0.968799	6.34381	0.975121	10.91652	0.986295
8	5.27723	0.964950	6.41056	0.971955	10.98350	0.984455
9	5.33599	0.961165	6.46954	0.968832	11.04263	0.982630
10^{-1}	5.38866	0.957438	6.52237	0.965748	11.09557	0.980817
1.5	5.59246	0.939502	6.72660	0.950805	11.29981	0.971918
2	5.73837	0.922486	6.87258	0.936496	11.44535	0.963239
3	5.94654	0.890501	7.08040	0.909306	11.65173	0.946393
4	6.09660	0.860645	7.22982	0.883626	11.79936	0.930105
5	6.21476	0.832509	7.34722	0.859185	11.91480	0.914288
6	6.31273	0.805843	7.44434	0.835817	12.00988	0.898891
7	6.39674	0.780472	7.52746	0.813405	12.09089	0.883877
8	6.47052	0.756264	7.60033	0.791862	12.16161	0.869217
9	6.53650	0.733117	7.66537	0.771117	12.22447	0.854892
1	6.59631	0.710945	7.72422	0.751115	12.28112	0.840882
1.5	6.83513	0.612514	7.95813	0.660739	12.50385	0.775099
2	7.01549	0.530765	8.13345	0.583641	12.66780	0.715541
3	7.29211	0.403392	8.39982	0.459397	12.91109	0.611804
4	7.51082	0.310190	8.60814	0.364725	13.09594	0.524834
5	7.69820	0.240609	8.78502	0.291451	13.24900	0.451383
6	7.86605	0.187937	8.94232	0.234120	13.38215	0.389032
7	8.02062	0.147648	9.08631	0.188895	13.50171	0.335899
8	8.16562	0.116577	9.22074	0.152986	13.61148	0.290481
9	8.30340	0.092451	9.34797	0.124320	13.71386	0.251559
10	8.43552	0.073609	9.46962	0.101332	13.81051	0.218129
15	9.04106	0.024700	10.02454	0.0378554	14.23985	0.108643
20	9.59075	0.00888347	10.52775	0.0148787	14.61988	0.055284

$\beta = \frac{Tt}{a^2}$	S=0.25	S=0.1		S=0.01		S=0.001		$S=10^{-4}$		$S=10^{-5}$		$S=10^{-6}$		$S=10^{-10}$	
	3	1	3	1	3	1	3	1	3	1	3	2	3	2	3
10^{-3}	0.9623	0.9771	0.9744	0.9920	0.9919	0.9969	0.9969	0.9985	0.9985	0.9992	0.9991	0.9994	0.9994	0.9997	0.9997
2	0.9485		0.9654		0.9882		0.9952		0.9975		0.9985	0.9989	0.9989	0.9995	0.9995
2.15		0.9658	0.9642	0.9876	0.9877	0.9949	0.9949	0.9974	0.9974	0.9985	0.9984				
4	0.9286		0.9519		0.9824							0.9980		0.9991	
4.64		0.9490	0.9483	0.9807	0.9809	0.9914	0.9915	0.9954	0.99538	0.9970	0.9969				
10^{-2}	0.8891	0.9238	0.9240	0.9693	0.9697	0.9853	0.9855	0.9915	0.9915	0.9942	0.9942	0.9956	0.9956	0.9978	0.9978
2	0.8453		0.8914		0.9534		0.9760		0.9851		0.9895	0.9919	0.9919	0.9958	0.9958
2.15		0.8860	0.8873	0.9505	0.9512	0.9744	0.9747	0.9841	0.9842	0.9888	0.9888				
4	0.7852		0.8444		0.9273		0.9598		0.9738		0.9810	0.9848	0.9849	0.9919	0.9919
4.64		0.8293	0.83199	0.9187	0.91997	0.9545	0.9550	0.9701	0.9704	0.9781	0.9782				
10^{-1}	0.6738	0.7460	0.7507	0.8655	0.8677	0.9183	0.9194	0.9434	0.94398	0.9572	0.9574	0.9655	0.9657	0.9807	0.9808
2	0.5623		0.648		0.7923		0.8636		0.9008		0.9225	0.9361	0.93650	0.9631	0.9632
2.15		0.6289	0.6362	0.7782	0.7825	0.8538	0.8560	0.8935	0.8947	0.9167	0.9175				
4	0.4294		0.5167		0.67961		0.7727		0.8267		0.8606	0.8828	0.8836	0.9298	0.9301
4.64		0.4782	0.4850	0.6436	0.6498	0.7436	0.7474	0.8031	0.8054	0.8410	0.8424				
1	0.2405	0.3117	0.3101	0.4598	0.4661	0.5729	0.5788	0.6520	0.6562	0.7080	0.7109	0.7489	0.7511	0.8401	0.8408
2	0.1167		0.1605		0.2769		0.3806		0.4644		0.5307	0.5800	0.5836	0.7139	0.7155
2.15		0.1665	0.1469	0.2597	0.2577	0.3543	0.3588	0.4364	0.4421	0.5038	0.5089				
4	0.0371		0.05532		0.1145		0.1823		0.2490		0.3102	0.3613	0.3647	0.5222	0.5248
4.64		0.07415	0.04084	0.1086	0.08856	0.1554	0.1465	0.2082	0.20638	0.2620	0.2634				
7	0.009133	0.04625	0.01467	0.06204	0.03661	0.08519	0.06850	0.1161	0.1067	0.1521	0.1476	0.1903	0.1889	0.3337	0.3358
10	0.0027517	0.03065	0.004647	0.03780	0.01324	0.04821	0.0281	0.06355	0.04867	0.08378	0.0736	0.1078	0.1013	0.2178	0.2181
14		0.02092	0.001196	0.02414	0.003883	0.02844	0.00937	0.03492	0.01827	0.04426	0.03059				
20	0.00010856		0.0002012		0.0007483		0.00209		0.004682		0.008883	0.02720	0.01488	0.06149	0.0553

Comparaison entre les résultats obtenus à partir de la solution analytique et publiés

par Cooper, Bredehoeft and Papadopulos, 1967 1
Papadopulos, Bredehoeft and Cooper, 1973 2.

et ceux que fournit la présente méthode. 3.


4. Slug test.
Nappe confinée. Ecoulement axisymétrique.
s_o/s_{oi} pour huit valeurs de S
s_o/s_{oi}
$\beta = \frac{Tt}{a^2}$
S=10^-10
S=10^-6
S=10^-5
S=10^-4
S=10^-3
S=10^-2
S=10^-1
S=0.25
1
10^-1
10^-2
10^-3
10^-4
10^-3
10^-2
10^-1
1
10
60

5.1.1. Milieu à double porosité. Nappe confinée. Ecoulement parallèle. Soutirage à niveau constant s_0.

Suivant Barenblatt and al (1960), un milieu à double porosité consiste en un assemblage de blocs poreux séparés par des fissures ou fractures. Les caractéristiques physiques des blocs seront notées 1 tandis que celles des fractures le seront par l'indice 2.

Continuité :
$$\begin{cases} \dfrac{d\dot{Q}_1}{dx} = S_1 \dfrac{ds_1}{dt} + \alpha_1 (s_1 - s_2) & (1) \\[2mm] \dfrac{d\dot{Q}_2}{dx} = S_2 \dfrac{ds_2}{dt} + \alpha_1 (s_2 - s_1) & (2) \end{cases}$$

Darcy :
$$\begin{cases} \dot{Q}_1 = T_1 \dfrac{ds_1}{dx} \\[2mm] \dot{Q}_2 = T_2 \dfrac{ds_2}{dx} \end{cases}$$

ou, en négligeant dans (1) et (2) le plus petit terme (voir T.D. Streltsova-Adams, 1978) : $\dfrac{d\dot{Q}_1}{dx} = S_2 \dfrac{ds_2}{dt}$ ou $\dfrac{dQ_1}{dx} = S_2 s_2$

De $S_2 \dfrac{ds_2}{dt} = \alpha_1 (s_1 - s_2)$ on tire : $s_2 = s_1 (1 - e^{-\varepsilon t})$ avec $\varepsilon = \dfrac{\alpha_1}{S_2}$ d'où :

$\dfrac{dQ_1}{dx} = S_2 s_1 (1 - e^{-\varepsilon t})$ On pose : $s_1 = s_0 \zeta^2$ avec $\zeta = 1 - \dfrac{x}{q}$

$$Q_1 = -S_2 s_{10} (1 - e^{-\varepsilon t}) \int_x^q \zeta^2 dx \quad \text{or } dx = -q\, d\zeta \text{ d'où :}$$

$$Q_1 = -S_2 s_{10} (1 - e^{-\varepsilon t})(-q) \int_\zeta^0 \zeta^2 d\zeta = -S_2 \frac{s_{10}}{3} (1 - e^{-\varepsilon t}) q \zeta^3$$

$$\frac{dQ_1}{dq} = -S_2 s_{10} (1 - e^{-\varepsilon t}) \frac{1}{3} \zeta^2 (3 - 2\zeta) \; ; \; \frac{dQ_1}{dt} = -S_2 \frac{s_{10}}{3} q \zeta^3 \varepsilon e^{-\varepsilon t} \text{ et } \dot{Q}_1 = \frac{\partial Q_1}{\partial q} \dot{q} + \frac{\partial Q_1}{\partial t}$$

$$\dot{Q}_1 = T_1 \frac{ds_1}{dx} \text{ d'où } \int_0^q \dot{Q}_1 \, \delta Q_1 \, dx = T_1 \int_0^q \frac{ds_1}{dx} \delta Q_1 \, dx \quad \text{or } \begin{cases} dx = -q\, d\zeta \\ \dfrac{ds_1}{dx} = -2 s_{10} \dfrac{\zeta}{q} \end{cases}$$

$$-q \int_0^1 \dot{Q}_1 \frac{\partial Q_1}{\partial q} d\zeta = -2 T_1 s_{10} \int_0^1 \zeta \frac{\partial Q_1}{\partial q} d\zeta \text{ ou } q\dot{q} \int_0^1 \left(\frac{\partial Q_1}{\partial q}\right)^2 d\zeta + q \int_0^1 \frac{\partial Q_1}{\partial t} \frac{\partial Q_1}{\partial q} d\zeta = -2 s_{10} T_1 \int_0^1 \zeta \frac{\partial Q_1}{\partial q} d\zeta$$

$$\text{or } q\dot{q} \int_0^1 \left(\frac{\partial Q_1}{\partial q}\right)^2 d\zeta = q\dot{q} S_2^2 s_{10}^2 (1 - e^{-\varepsilon t}) \frac{1}{9} \int_0^1 \zeta^4 (3 - 2\zeta)^2 d\zeta = \frac{13}{9 \times 35} q\dot{q} S_2^2 s_{10}^2 (1 - e^{-\varepsilon t})^2$$

$$q \int_0^1 \frac{\partial Q_1}{\partial t} \frac{\partial Q_1}{\partial q} d\zeta = q S_2^2 \frac{s_{10}^2}{9} q \varepsilon e^{-\varepsilon t} (1 - e^{-\varepsilon t}) \int_0^1 \zeta^3 \zeta^2 (3 - 2\zeta) d\zeta = \frac{3}{9 \times 14} q S_2^2 s_{10}^2 q \varepsilon e^{-\varepsilon t} (1 - e^{-\varepsilon t})$$

$$-2 s_{10} T_1 \int_0^1 \zeta \frac{\partial Q_1}{\partial q} d\zeta = -2 s_{10}^2 T_1 (-S_2)(1 - e^{-\varepsilon t}) \frac{1}{3} \int_0^1 \zeta \zeta^2 (3 - 2\zeta) d\zeta = -\frac{7}{30} T_1 (-S_2) s_{10}^2 (1 - e^{-\varepsilon t})$$

$$\text{d'où } \frac{13}{105} (1 - e^{-\varepsilon t}) q\dot{q} + \frac{1}{14} q^2 \varepsilon e^{-\varepsilon t} = \frac{7}{10} \frac{T_1}{S_2} \text{ ou } \frac{13}{15} (1 - e^{-\varepsilon t}) \frac{dq^2}{dt} + \varepsilon e^{-\varepsilon t} q^2 = \frac{49}{5} \frac{T_1}{S_2}$$

Soit $\dfrac{T_1 t}{S_2} = \alpha$ et $\varepsilon t = \eta \alpha$ il vient : $(1 - e^{-\eta\alpha}) \dfrac{dq^2}{d\alpha} + \dfrac{15}{13} \eta e^{-\eta\alpha} q^2 = \dfrac{147}{13}$

Pour $\alpha \to \infty$, on retrouve la formule valable en milieux homogène : $q^2 = \frac{147}{13}\alpha$ (Problème 1.1.1.) On commencera l'intégration de l'équation différentielle par la valeur $\alpha = 10^6$ et $q = 3362.691230$, avec un pas négatif.

$\dot{Q}_{10} = \frac{\partial Q_{10}}{\partial q}\dot{q} + \frac{\partial Q_{10}}{\partial t}$ devient :

$$\dot{Q}_{10} = -S_2\frac{s_{10}}{3}(1-e^{-\varepsilon t})\left[\frac{147}{26}\frac{T_1}{S_2 q(1-e^{-\varepsilon t})} - \frac{15}{26}\frac{q\varepsilon e^{-\varepsilon t}}{1-e^{-\varepsilon t}}\right] - S_2\frac{s_{10}}{3}q\,e^{-\varepsilon t}$$

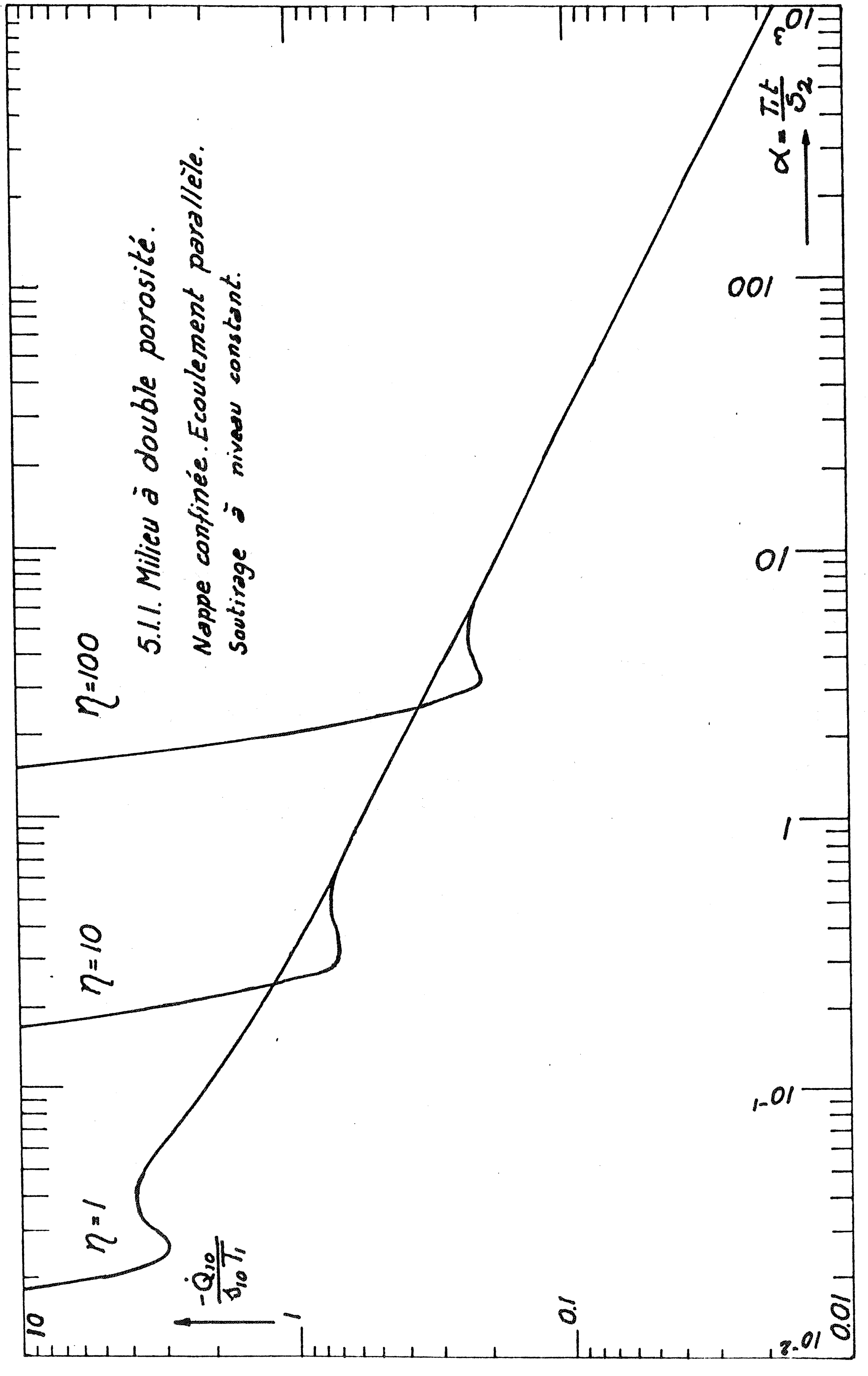
$$= -\frac{S_2 s_{10}}{3}\left[\frac{147}{26}\frac{T_1}{S_2 q} - \frac{15}{26}q\varepsilon e^{-\varepsilon t} + q\varepsilon e^{-\varepsilon t}\right] = \frac{-S_2 s_{10}}{3\times 26}\left[147\frac{T_1}{S_2 q} + 11 q\varepsilon e^{-\varepsilon t}\right] \text{ ou}$$

$$-\frac{\dot{Q}_{10}}{s_{10}T_1} = \frac{49}{26q} + \frac{11}{78}S_2 q\varepsilon e^{-\varepsilon t} \text{ ou enfin: } \frac{-\dot{Q}_{10}}{s_{10}T_1} = \frac{49}{26}\frac{1}{q} + \frac{11}{78}q\,\eta\,e^{-\eta\alpha}$$

α	$\eta = 1$		$\eta = 10$		$\eta = 100$	
	q	$-\dot{Q}_{10}/s_{10}T_1$	q	$-\dot{Q}_{10}/s_{10}T_1$	q	$-\dot{Q}_{10}/s_{10}T_1$
10^6	3362.6912	0.0005604				
$9\ 10^5$	3190.1290	0.0005907				
8	3007.6824	6268				
7	2813.4293	6698				
6	2604.7294	7235				
5	2377.7817	7926				
4	2126.7526	8861				
3	1841.8217	0.0010232				
2	1503.8411	12532				
1.5	1302.3646	14470				
10^5	1063.3762	17723				
$9\ 10^4$	1008.8072	18681				
8	951.1125	19814				
7	889.6843	21183				
6	823.6875	22880				
5	751.9204	25064				
4	672.5380	28022				
3	582.4349	32357				
2	475.5559	39629				
1.5	411.8434	45760				
10^4	336.2685	56045				
$9\ 10^3$	319.0123	59076				
8	300.7676	62660				
7	281.3423	66986				
6	260.4722	72353				
5	237.7774	79259				
4	212.6744	88615				
3	184.1812	0.0102323				
2	150.3829	125321				
1.5	130.2350	144708				
10^3	106.3359	177232				

valeurs identiques à celles obtenues pour $\eta = 1$ puisque $e^{-\eta\alpha} \to 0$

	$\eta = 1$		$\eta = 10$		$\eta = 100$	
10^3	106.3359	0.017723	valeurs identiques à celles obtenues pour $\eta = 1$		valeurs identiques à celles obtenues pour $\eta = 10$	
$9\ 10^2$	100.8789	0.018682				
8	95.1093	0.019815				
7	88.9664	0.021183				
6	82.3665	0.022880				
5	75.1896	0.025064				
4	67.2511	0.028023				
3	58.2404	0.032359				
2	47.5518	0.039632				
1.5	41.1799	0.045765				
10^2	33.6214	0.056053				
9.10	31.8955	0.059087				
8	30.0707	0.062672				
7	28.1278	0.067001				
6	26.0403	0.072373				
5	25.7701	0.079285				
4	21.2589	0.088650				
3	18.4083	0.102378				
2	15.0263	0.125421	15.0263	0.125421		
1.5	13.0097	0.144862	13.0096	0.144862		
10	10.6245	0.177451	10.6166	0.177515		
9 . 1.	10.0906	0.186943	10.0700	0.187151		
8	9.5456	0.197883	9.49197	0.198548		
7	9.0156	0.210198	8.87636	0.212318		
6	8.5766	0.222735	8.214749	0.229418		
5	8.4492	0.231080	7.494959	0.251451		
4	9.3473	0.225763	6.698263	0.281358		
3	14.5192	0.231744	5.793016	0.325325	5.793016	0.325325
2	59.0511	1.15895	4.717133	0.399525	4.717133	0.399525
1.5	294.1137	9.26130	4.074022	0.462595	4.074002	0.462595
1	6462.75	335.2909	3.310598	0.569479	3.308118	0.569694
0.9	16662.92	955.3977	3.139003	0.600932	3.132551	0.601623
0.8	52127.60	3303.1649	2.963280	0.637391	2.946542	0.639602
0.7	196138.94	13735.85	2.791310	0.678762	2.747970	0.685820
0.6	1010815.367	78233.58	2.646307	0.721418	2.533884	0.743765
0.5	7811243.187	668145.42	2.595694	0.750719	2.299957	0.819413
0.4			2.857772	0.733285	2.039371	0.924116
0.3			4.423191	0.736639	1.740191	1.082993
0.2			17.972433	3.53503	1.377496	1.368145
0.15			89.512258	28.1879	1.154176	1.632870
0.1	$1.27\ 10^{16}$	$1.63\ 10^{15}$	1966.91088	1020.4428	0.876310	2.151185
0.09			5071.2851	2907.71	0.810234	2.327422
8			15560.4569	9860.18	0.739695	2.551324
7			59694.0091	41804.4	0.665139	2.841966
6			307637.1	238100.4	0.589346	3.218407
5			2377316.8	2033470.6	0.521968	3.660189
4			33716779.7	31873252.1	0.496190	3.926332
3					0.663489	3.306312
2					2.570117	5.638540
15					12.782791	40.37114
0.01			$3.89\ 10^{15}$	$4.96\ 10^{15}$	280.87408	1457.1936
0.009					724.1774	4152.2004
0.008					2222.026	14080.3
0.007					8524.280	59696.6


5.1.1. Milieu à double porosité.
Nappe confinée. Ecoulement parallèle.
Soutirage à niveau constant.
η=100
η=10
η=1
$\frac{-\dot{Q}_{10}}{s_{10} T_1}$
$\alpha = \frac{T_1 t}{S_2}$
10
1
0.1
0.01
10^{-2}
10^{-1}
1
10
100
10^3

5.1.2. Milieu à double porosité. Nappe confinée. Ecoulement parallèle. Soutirage à débit constant : $-\dot{Q}_{01}$.

Continuité : $\dfrac{d\dot{Q}_1}{dx} = S_1 \dfrac{ds_1}{dt} + \alpha_1 (s_1 - s_2)$

$\dfrac{d\dot{Q}_2}{dx} = S_2 \dfrac{ds_2}{dt} + \alpha_1 (s_2 - s_1)$

Darcy : $\dot{Q}_1 = T_1 \dfrac{ds_1}{dx}$

$\dot{Q}_2 = T_2 \dfrac{ds_2}{dx}$ ou, suivant les hypothèses de Barenblatt and al.

$\dfrac{d\dot{Q}_1}{dx} = S_2 \dfrac{ds_2}{dt}$ ou $\dfrac{dQ_1}{dx} = S_2 s_2$ or $S_2 \dot{s}_2 = \alpha_1 (s_1 - s_2)$ d'où $s_2 = s_1 (1 - e^{-\varepsilon t})$ avec $\varepsilon = \dfrac{\alpha_1}{S_2}$

$\dfrac{dQ_1}{dx} = S_2 s_1 (1 - e^{-\varepsilon t})$. On pose $s_1 = s_{10} \zeta^2$ avec $\zeta = 1 - \dfrac{x}{q}$ d'où

$Q_1 = -S_2 s_{10} (1 - e^{-\varepsilon t}) \displaystyle\int_x^q \zeta^2 dx$ or $dx = -q\, d\zeta$ d'où :

$$Q_1 = -S_2 s_{10} (1 - e^{-\varepsilon t}) \int_0^\zeta \zeta^2 d\zeta = -S_2 \frac{s_{10}}{3} (1 - e^{-\varepsilon t}) q \zeta^3$$

$$\frac{\partial Q_1}{\partial q} = -S_2 \frac{s_{10}}{3} (1 - e^{-\varepsilon t}) \left[\zeta^3 + 3q\zeta^2 \frac{d\zeta}{dq}\right] = -S_2 \frac{s_{10}}{3} (1 - e^{-\varepsilon t}) \zeta^2 (3 - 2\zeta) \quad \text{vu } \frac{d\zeta}{dq} = \frac{x}{q^2} = \frac{1-\zeta}{q}$$

$$\frac{\partial Q_1}{\partial s_{10}} = -S_2 \frac{1}{3} (1 - e^{-\varepsilon t}) \zeta^3 q$$

$$\frac{\partial Q_1}{\partial t} = -S_2 \frac{s_{10}}{3} q \zeta^3 \varepsilon e^{-\varepsilon t} \qquad \text{et } \dot{Q}_1 = \frac{\partial Q_1}{\partial q} \dot{q} + \frac{\partial Q_1}{\partial s_{10}} \dot{s}_{10} + \frac{\partial Q_1}{\partial t}$$

Darcy : $\dot{Q}_1 = T_1 \dfrac{ds_1}{dx}$ ou $\displaystyle\int_0^q \dot{Q}_1 \delta Q_1 dx = T_1 \int_0^q \frac{ds_1}{dx} \delta Q_1 dx$ ou $\displaystyle\int_0^q \dot{Q}_1 \frac{\partial Q_1}{\partial q} dx = T_1 \int_0^q \frac{ds_1}{dx} \frac{\partial Q_1}{\partial q} dx$

ou, puisque $dx = -q\, d\zeta$ et $\dfrac{ds_1}{dx} = -2 s_{10} \dfrac{\zeta}{q}$

$$\int_0^1 \left(\frac{\partial Q_1}{\partial q} \dot{q} + \frac{\partial Q_1}{\partial s_{10}} \dot{s}_{10} + \frac{\partial Q_1}{\partial t}\right) \frac{\partial Q_1}{\partial q} d\zeta = -2T_1 \frac{s_{10}}{q} \int_0^1 \zeta \frac{\partial Q_1}{\partial q} d\zeta \quad \text{ou } A + B + C = D.$$

Calcul de $A = \dot{q} \displaystyle\int_0^1 \left(\frac{\partial Q_1}{\partial q}\right)^2 d\zeta = \dot{q} (-S_2)^2 \frac{s_{10}^2}{3.3} (1 - e^{-\varepsilon t})^2 \underbrace{\int_0^1 \zeta^4 (3 - 2\zeta)^2 d\zeta}_{= 13/35}$

Calcul de $B = \dot{s}_{10} \displaystyle\int_0^1 \left(\frac{\partial Q_1}{\partial s_{10}}\right)\left(\frac{\partial Q_1}{\partial q}\right) d\zeta = \dot{s}_{10} (-S_2)^2 \frac{s_{10}}{3.3} (1 - e^{-\varepsilon t})^2 q \underbrace{\int_0^1 \zeta^3 \zeta^2 (3 - 2\zeta) d\zeta}_{3/14}$

Calcul de $C = \displaystyle\int_0^1 \frac{\partial Q_1}{\partial t} \frac{\partial Q_1}{\partial q} d\zeta = (-S_2)^2 \frac{s_{10}}{3.3} q \varepsilon e^{-\varepsilon t} (1 - e^{-\varepsilon t}) \underbrace{\int_0^1 \zeta^3 \zeta^2 (3 - 2\zeta) d\zeta}_{3/14}$

Calcul de $D = -2T_1 \dfrac{s_{10}}{q} (-S_2) \dfrac{s_{10}}{3} (1 - e^{-\varepsilon t}) \underbrace{\displaystyle\int_0^1 \zeta^3 (3 - 2\zeta) d\zeta}_{7/20}$ d'où :

$$\frac{13}{15} \frac{\dot{q}}{q} + \frac{1}{2} \frac{\dot{s}_{01}}{s_{01}} + \frac{1}{2} \frac{\varepsilon e^{-\varepsilon t}}{1 - e^{-\varepsilon t}} = \frac{49}{10} \frac{T_1}{S_2} \frac{1}{q^2 (1 - e^{-\varepsilon t})} \qquad (')$$

La contrainte $\dot{Q}_{1e} t = Q_{10}$ devient :

$$\frac{1}{t} = \frac{\dot q}{q} + \frac{\dot\delta_{10}}{\delta_{10}} + \frac{\varepsilon e^{-\varepsilon t}}{1-e^{-\varepsilon t}}$$ d'où (') devient :

$$\frac{13}{15}\frac{\dot q}{q} + \frac{1}{2}\left[\frac{1}{t} - \frac{\dot q}{q}\right] = \frac{49}{10}\frac{T_1}{S_2}\frac{1}{q^2(1-e^{-\varepsilon t})}$$ ou $$\frac{11}{30}\frac{\dot q}{q} + \frac{1}{2t} = \frac{49}{10}\frac{T_1}{S_2}\frac{1}{q^2(1-e^{-\varepsilon t})}$$

Soit $\alpha = \frac{T_1 t}{S_2}$ et $\eta\alpha = \varepsilon t$ d'où :

$$\frac{11}{15}\frac{dq}{d\alpha} + \frac{q}{\alpha} = \frac{49}{5}\frac{1}{q}\frac{1}{1-e^{-\eta\alpha}}$$ Equation linéaire en q^2, dont l'intégration est difficile aussi peut-on raisonner autrement. Pour $\alpha \to 0$, il vient :

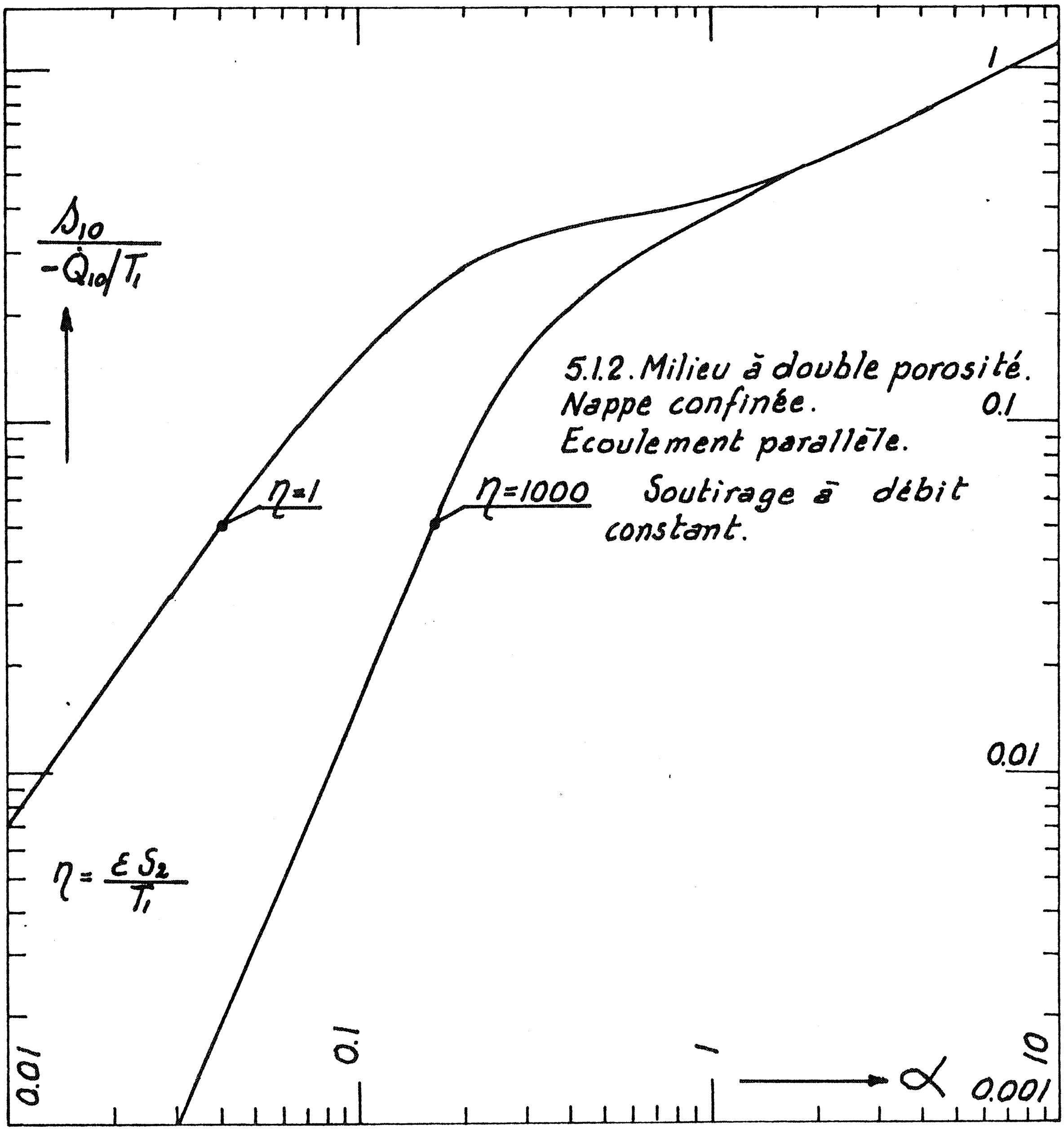
$$\frac{dq^2}{d\alpha} = \frac{2}{11\alpha}\left(\frac{147}{\eta} - 15q^2\right)$$ dont la solution est : $$q^2 = \frac{147}{15\eta} + \frac{1}{15}\alpha^{-\frac{30}{11}}$$ d'où :

pour $\alpha = 10^{-5}$ et $\eta = 1$ ou 1000, $q = 1698\,776.269$

$$Q_{10} = -S_2\frac{\dot\delta_{10}}{3}(1-e^{-\eta\alpha})q$$ d'où $$\frac{\dot\delta_{10}}{\left(-\frac{\dot Q_{10}}{T_1}\right)} = \frac{\alpha}{q(1-e^{-\eta\alpha})} \begin{cases} = \frac{1}{\eta q} & \text{pour } \alpha \text{ petit} \\ = \frac{\alpha}{q} & \text{"} \quad \text{" grand} \end{cases}$$

α	$\eta = 1$		$\eta = 1000$	
	q	$\dot\delta_{10}/-\frac{\dot Q_{10}}{T_1}$	q	$\dot\delta_{10}/-\frac{\dot Q_{10}}{T_1}$
10^{-5}	1698 776.269	0.000000 589	1698 776.269	10^{-9}
1.5	977 264.2307	0.000001 023	977 264.	
2	660 147.5897	1515	660 147.	
3	379 766.6816	2633	379 766.	
4	256 534.5703	3898	256 534.	
5	189 232.5721	5285	189 232.	5.10^{-9}
6	147 578.0052	6776	147 578.	
7	119 599.8489	8361	119 599.	
8	99 689.8030	0.000010032	99 689.	
9	84 897.9721	11779	84 897.	
10^{-4}	73 536.1264	13599	73 536.	$1.4\ 10^{-8}$
1.5	42 303.5260	23640	42 303.	
2	28 576.2744	34998	28 576.	
3	16 439.2284	60839	16 439.	
4	11 104.7933	90069	11 104.	
5	8 191.4443	0.000 122109	8191.	$16.5\ 10^{-8}$
6	6388.3139	156 583	6388.	
7	5177.2039	193 222	5177.	
8	4315.3439	231 824	4315.	
9	3675.0397	272 228	3675.	
10^{-3}	3183.2117	314 305	3183.	$99.2\ 10^{-8}$
1.5	1831.2252	546492	1831.	$1.054\ 10^{-6}$
2	1237.0053	809213	1237.	1.870 "
3	711.6233	0.00 140734	711.	4.437 "
4	480.7112	208441	480.	8.476 "
5	354.6026	282711	354.	14.196 "
6	276.5531	362680	276.	21.751 "

7	224.1311	447730	224.	31.263 "
8	186.8276	537396	186	42.841 "
9	159.1148	631309	159.	56.581 "
10^{-2}	137.8295	729167	137	72.575 "
1.5	79.3315	0.01270010	79.270	189.227 "
2	53.6390	1883018	53.548	0.000 373495
3	30.9646	3278173	30.807	0.000 973781
4	21.0460	4847152	20.815	0.001921660
5	15.6710	6542042	15.361	0.003254994
6	12.3819	8321007	11.988	0.005004779
7	10.2061	0.10145	9.727	0.007136455
8	8.6877	0.11977	8.121	0.008850284
9	7.5861	0.13784	6.933	0.012981243
10^{-1}	6.7632	0.15537	6.024	0.016598641
1.5	4.7035	0.22895	3.584	0.041844
2	3.9885	0.27662	2.609	0.076657
3	3.5631	0.32485	1.982	0.151343
4	3.4814	0.34850	1.918	0.208524
5	3.4859	0.36453	2.006	0.2491
6	3.5197	0.37781	2.138	0.2806
7	3.5659	0.38994	2.279	0.3071
8	3.6180	0.401537	2.420	0.3305
9	3.6733	0.412863	2.557	0.3518
1	3.7306	0.424048	2.690	0.3717
1.5	4.0290	0.479224	3.283	0.4569
2	4.3339	0.533701	3.788	0.5279
3	4.9411	0.638953	4.638	0.6467
4	5.5301	0.736800	5.355	0.7468
5	6.0920	0.826309	5.987	0.8350
6	6.6240	0.908047	6.559	0.9147
7	7.1266	0.983130	7.084	0.9880
8	7.6020	1.052704	7.574	1.0562
9	8.0529	1.117745	8.033	1.1203
10	8.4819	1.179024	8.468	1.1809
1.5	10.3749	1.445790		
2	11.9770	1.669855		
3	14.6674	2.045347		
4	16.9362	2.361806		
50	18.9351	2.640593		



$\frac{s_{10}}{-\dot{Q}_{10}/T_1}$
5.1.2. Milieu à double porosité.
Nappe confinée.
Ecoulement parallèle.
Soutirage à débit constant.
$\eta=1$
$\eta=1000$
$\eta = \frac{\varepsilon S_2}{T_1}$
α
1
0.1
0.01
0.001
0.01
0.1
1
10

5.2.1. Milieu à double porosité. Nappe confinée. Ecoulement axisymétrique. Soutirage à niveau constant.

Continuité : $\dfrac{dQ_1}{dr} = 2\pi r S_1 \dfrac{ds_1}{dt} + \alpha_1 (s_1 - s_2)\, 2\pi r$

$\dfrac{dQ_2}{dr} = 2\pi r S_2 \dfrac{ds_2}{dt} + \alpha_2 (s_2 - s_1) 2\pi r$

Darcy : $\dot{Q}_1 = 2\pi r T_1 \dfrac{ds_1}{dr}$

$\dot{Q}_2 = 2\pi r T_2 \dfrac{ds_2}{dr}$ ou, avec les approximations déja faites plus haut

$\dfrac{d\dot{Q}_1}{dr} = 2\pi r S_2 \dfrac{ds_2}{dt}$ ou $\dfrac{dQ_1}{dr} = 2\pi r S_2 s_2$. De $2\pi r S_2 \dfrac{ds_2}{dt} = 2\pi r \alpha_1 (s_1 - s_2)$ on tire $s_2 = s_1 (1 - e^{-\varepsilon t})$ avec $\varepsilon = \dfrac{\alpha_1}{S_2}$

On pose : $s_1 = s_{10} \dfrac{u_1}{u_{10}}$ avec $u_1 = \log \dfrac{R_1}{r}$ et $u_{10} = \log \dfrac{R_1}{a}$

d'où $\dfrac{dQ_1}{dr} = 2\pi r S_2 s_{10} \dfrac{u_1}{u_{10}} (1 - e^{-\varepsilon t})$

$Q_1 = -2\pi S_2 s_{10} (1 - e^{-\varepsilon t}) \dfrac{1}{u_{10}} \displaystyle\int_r^R r u_1 dr$ or $r dr = -\dfrac{a^2}{e^{-2u_{10}}} e^{-2u_1} du_1$ d'où

$$Q_1 = (-2\pi S_2) s_{10} (1 - e^{-\varepsilon t}) \frac{1}{u_{10}} \left(\frac{-a^2}{e^{-2u_{10}}} \right) \underbrace{\int_{u_1}^{0} u_1 e^{-2u_1} du_1}_{-\frac{1}{4}\varphi_1}$$

$$\frac{\partial Q_1}{\partial u_{10}} = (-2\pi S_2) s_{10} (1 - e^{-\varepsilon t}) \frac{a^2}{4u_{10}^2 e^{-2u_{10}}} \left[u_{10} \frac{\partial \varphi_1}{\partial u_1} + (2u_{10} - 1) \varphi_1 \right]$$

$\dfrac{\partial Q_1}{\partial t} = (-2\pi S_2) s_{10} \dfrac{a^2}{4 u_{10} e^{-2u_{10}}} \varphi_1 \varepsilon e^{-\varepsilon t}$ et $\dot{Q}_1 = \dfrac{\partial Q_1}{\partial u_{10}} \dot{u}_{10} + \dfrac{\partial Q_1}{\partial t}$

Darcy : $\dot{Q}_1 = 2\pi r T_1 \dfrac{ds_1}{dr} = -2\pi T_1 \dfrac{s_{10}}{u_{10}}$ d'où $\displaystyle\int_a^R \dot{Q}_1 \delta Q_1 dr = -2\pi T_1 \frac{s_{10}}{u_{10}} \int_a^R \delta Q_1 dr$ ou

$$\int_0^{u_1} \dot{Q}_1 \frac{\partial Q_1}{\partial u_{10}} du_1 = -2\pi T_1 \frac{s_{10}}{u_{10}} \int_0^{u_1} \frac{\partial Q_1}{\partial u_{10}} du_1, \text{ ou } \int_0^{u_1} \left(\frac{\partial Q_1}{\partial u_1} \right)^2 \dot{u}_{10} du_1 + \int_0^{u_1} \frac{\partial Q_1}{\partial t} \frac{\partial Q_1}{\partial u_{10}} du_1 = -2\pi T_1 \frac{s_{10}}{u_{10}} \int_0^{u_1} \frac{\partial Q_1}{\partial u_1} du_1 \quad (')$$

$$\text{or } \dot{u}_{10} \int_0^{u_1} \left(\frac{\partial Q_1}{\partial u_{10}} \right)^2 du_1 = \dot{u}_{10} (-2\pi S_2)^2 s_{10}^2 (1 - e^{-\varepsilon t})^2 \left(\frac{a^2}{4u_{10}^2 e^{-2u_{10}}} \right)^2 \underbrace{\int_0^{u_1} \left[u_{10} \frac{\partial \varphi_1}{\partial u_1} + (2u_{10} - 1) \varphi_1 \right]^2 du_1}_{I_1}$$

$$\int_0^{u_1} \frac{\partial Q_1}{\partial u_1} \frac{\partial Q_1}{\partial t} du_1 = (-2\pi S_2)^2 s_{10}^2 (1 - e^{-\varepsilon t}) \left(\frac{a^2}{4u_{10}^2 e^{-2u_{10}}} \right)^2 u_{10} \varepsilon e^{-\varepsilon t} \underbrace{\int_0^{u_1} \varphi_1 \left[u_{10} \frac{\partial \varphi_1}{\partial u_1} + (2u_{10} - 1) \varphi_1 \right] du_1}_{I_2}$$

et $-2\pi T_1 \dfrac{s_{10}}{u_{10}} (-2\pi S_2) s_{10} (1 - e^{-\varepsilon t}) \dfrac{a^2}{4u_{10}^2 e^{-2u_{10}}} \underbrace{\displaystyle\int_0^{u_1} \left[u_{10} \frac{\partial \varphi_1}{\partial u_1} + (2u_{10} - 1) \varphi_1 \right] du_1}_{I_4}$ et (') devient :

$$S_2\left(\frac{a^2}{4u_{10}\,e^{-2u_{10}}}\right)\left[(1-e^{-\varepsilon t})\,I_1\,\dot{u}_{10}+u_{10}\,\varepsilon e^{-\varepsilon t} I_2\right]=T_1 I_4$$ ou, enfin :

$$(1-e^{-\varepsilon t})\frac{I_1}{4I_4}\frac{\dot{u}_{10}}{u_{10}}+\varepsilon e^{-\varepsilon t}\frac{I_2}{4I_4}=\frac{T_1}{S_2 a^2}\,e^{-2u_{10}}$$

Soit $\alpha=\frac{T_1 t}{S_2 a^2}$ et $\varepsilon t=\eta\alpha$ avec $\eta=\frac{a^2\alpha_1}{T_1}$ d'où $(1-e^{-\eta\alpha})\frac{I_1}{u_{10}}\frac{du_{10}}{d\alpha}+\eta e^{-\eta\alpha}I_2=e^{-2u_{10}}\,4\,I_4$

ou $$\boxed{\frac{du_{10}}{d\alpha}=\frac{4u_{10}e^{-2u_{10}}I_4-\eta\, I_2\, u_{10}\,e^{-\eta\alpha}}{I_1(1-e^{-\eta\alpha})}}$$ (2) Pour $\alpha\to\infty$, on retrouve la formule valable en milieu homogène (1.2.1.)

Si $\alpha\to 0$ en même temps que $u_{10}\to\infty$, il vient : $\frac{du_{10}}{d\alpha}=\frac{-\eta(1-\eta\alpha)(2u_{10}^2+\cdots)u_{10}}{(4u_{10}^3+\cdots)\eta\alpha}$ ou

$\frac{1-\eta\alpha}{\alpha}\,d\alpha=-2\,du_{10}$ ou $u_{10}=\frac{1}{2}(\eta\alpha-\log\alpha)+C$

Pour $\alpha=10^{-5}$, et $\eta=1$; $u_{10}=5.756467730$
10 ; $u_{10}=5.756512730$
100 ; $u_{10}=5.756962730$
1000 ; $u_{10}=5.761462730$

$\dot{Q}_{10}=\frac{\partial Q_{10}}{\partial u_{10}}\,\dot{u}_{10}=\frac{\partial Q_{10}}{\partial t}$ devient

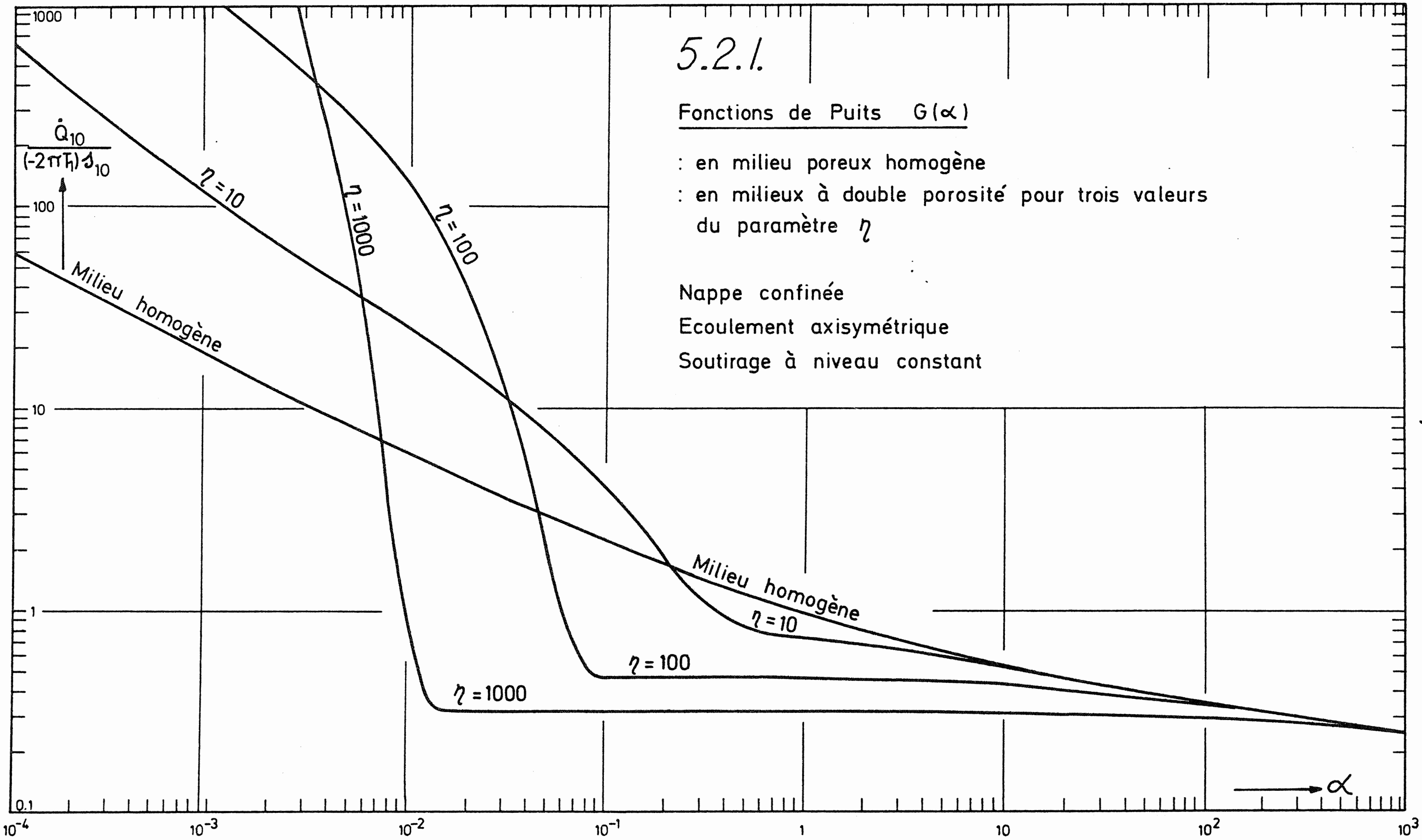
$$\dot{Q}_{10}=(-2\pi S_2)\,s_{10}\,(1-e^{-\varepsilon t})\frac{a^2}{4u_{10}^2\,e^{-2u_{10}}}\,\psi_{10}\,\dot{u}_{10}+(-2\pi S_2)\,s_{10}\frac{a^2}{4u_{10}^2\,e^{-2u_{10}}}\,u_{10}\,\varphi_{10}\,\varepsilon e^{-\varepsilon t}$$

$$=(-2\pi S_2)\,s_{10}\frac{a^2}{4u_{10}e^{-2u_{10}}}\left[\psi_{10}\frac{\frac{T_1}{S_2a^2}e^{-2u_{10}}-\varepsilon e^{-\varepsilon t}\frac{I_2}{4I_4}}{\frac{I_1}{4I_4}}+\varphi_{10}\,\varepsilon e^{-\varepsilon t}\right]$$ d'où

$$\frac{\dot{Q}_{10}}{(-2\pi T_1)s_{10}}=\frac{\psi_{10}}{u_{10}}\frac{I_4}{I_1}+\frac{\varepsilon e^{-\varepsilon t}}{\left(\frac{T_1}{S_2a^2}\right)4e^{-2u_{10}}}\frac{F_0}{I_1}$$ et si $\varepsilon t=\eta\alpha$

$$\boxed{\frac{\dot{Q}_{10}}{(-2\pi T_1)s_{10}}=\frac{\psi_{10}}{u_{10}}\frac{I_4}{I_1}+\frac{\eta e^{-\eta\alpha}}{4e^{-2u_{10}}}\frac{F_0}{I_1}}$$

α	$\eta=10$		$\eta=100$		$\eta=1000$		$\eta=\infty$ milieu homogène	
	u_{10}	$\dot{Q}_{10}/2\pi T_1 \delta_{10}$	u_{10}	$\dot{Q}_{10}/2\pi T_1 \delta_{10}$	u_{10}	$\dot{Q}_{10}/2\pi T_1 \delta_{10}$	u_{10}	$\dot{Q}_{10}/2\pi T_1 \delta_{10}$
10^{-3}	3.466	112.41	3.488	1056.71	3.691	5720.9	0.06849	18.1468
2	3.130	70.222	3.173	619.016	3.537	1693.1		
3	2.937	53.620	3.000	442.812	3.491	583.94		
5	2.698	38.324	2.799	277.147	3.469	78.932	(Probl.	1.2.1)
7	2.544	30.749	2.682	194.240	3.46643	10.637		
10^{-2}	2.384	24.323	2.574	124.949	3.4660131	0.8232	0.20276	6.0644
2	2.091	15.232	2.427	38.213	3.4660153	0.3093		
3	1.933	11.389	2.383	13.577	3.466039	0.30929		
5	1.755	7.6317	2.363	2.1941	3.466088	0.3092902		
7	1.655	5.6822	2.361	0.6989	3.46613	0.309285		
10^{-1}	1.566	4.0067	2.361	0.4772	3.46621	0.309278	0.5396	2.2222
2	1.460	1.8215	2.364	0.4650	3.46645	0.309255		
3	1.442	1.1632	2.366	0.4645	3.46669	0.309232		
5	1.462	0.8295	2.371	0.4635	3.4671	0.309186		
7	1.495	0.7667	2.376	0.4625	3.4676	0.309140		
1	1.542	0.7349	2.383	0.4610	3.468	0.309071	1.1863	0.9717
2	1.671	0.6741	2.406	0.4563	3.4708	0.308841		
3	1.771	0.6335	2.428	0.4519	3.4732	0.308614		
5	1.922	0.5805	2.469	0.4438	3.4780	0.308162		
7	2.035	0.5460	2.507	0.4367	3.4827	0.307717		
10	2.164	0.5110	2.558	0.4273	3.4898	0.307059	2.0986	0.5280
2	2.441	0.4493	2.698	0.4037	3.5125	0.304952		
3	2.614	0.4177	2.806	0.3872	3.5342	0.302965		
5	2.840	0.3823	2.968	0.3647	3.5749	0.299309		
7	2.992		3.089	0.3495	3.6125	0.296012		
10^{2}	3.156	0.34157	3.227	0.3336	3.6639	0.291616	3.1493	0.3424
2	3.482		3.519	0.3043	3.8041	0.280251		
3	3.674	0.2907	3.700	0.2886	3.9129	0.272025		
5	3.919		3.934	0.2704	4.0768	0.260485		
7	4.081		4.092		4.1994	0.252476		
10^{3}	4.254	0.2490	4.262	0.2485	4.3397	0.243882	4.2532	0.2491
2			4.595	0.2296	4.6359	0.227526		
3			4.791	0.2197	4.8193	0.218447		
5			5.040		5.0573	0.207686		
7			5.205		5.2171	0.201036		
10^{4}			5.379	0.19469	5.3883	0.194366	5.3788	0.19472
2					5.7246	0.182466		
3					5.9229	0.176108		
5					6.1737	0.168672		
7					6.3394	0.164095		
10^{5}					6.5152	0.159498	6.5143	0.1595
2					6.8577	0.151248		
3					7.0583	0.146798		
5					7.3114	0.141546		
7					7.4781	0.138285		
10^{6}					7.6550	0.134986	7.6549	0.13499


5.2.1.
Fonctions de Puits G(α)
: en milieu poreux homogène
: en milieux à double porosité pour trois valeurs du paramètre η
Nappe confinée
Ecoulement axisymétrique
Soutirage à niveau constant
Q̇10 / (-2πT1) s10
1000
100
10
1
0.1
η = 10
η = 100
η = 1000
Milieu homogène
Milieu homogène
η = 10
η = 100
η = 1000
α
10^-4
10^-3
10^-2
10^-1
1
10
10^2
10^3

5.2.2. Milieu à double porosité. Nappe confinée. Ecoulement axisymétrique. Soutirage à débit constant : $-\dot{Q}_{10}$.

$$\frac{dQ_1}{dr} = 2\pi r S_2 \frac{s_{10}}{u_{10}} u_1 (1-e^{-\varepsilon t})$$

$$Q_1 = -2\pi S_2 s_{10} (1-e^{-\varepsilon t}) \frac{1}{u_{10}} \int_r^R r u_1 dr = (-2\pi S_2) s_{10} (1-e^{-\varepsilon t}) \frac{a^2 \varphi_1 u_{10}}{4u_{10}^2 e^{-2u_{10}}}$$

$$\frac{\partial Q_1}{\partial u_{10}} = (-2\pi S_2) s_{10} (1-e^{-\varepsilon t}) \frac{a^2}{4u_{10}^2 e^{-2u_{10}}} \left[u_{10} \frac{\partial \varphi_1}{\partial u_1} + (2u_{10}-1)\varphi_1\right]$$

$$\frac{\partial Q_1}{\partial s_{10}} = (-2\pi S_2)(1-e^{-\varepsilon t}) \frac{a^2}{4u_{10}^2 e^{-2u_{10}}} u_{10} \varphi_1$$

$$\frac{\partial Q_1}{\partial t} = (-2\pi S_2) s_{10} \frac{a^2}{4u_{10}^2 e^{-2u_{10}}} u_{10} \varphi_1 \varepsilon e^{-\varepsilon t} \quad \text{et} \quad \dot{Q}_1 = \frac{\partial Q_1}{\partial u_{10}} \dot{u}_{10} + \frac{\partial Q_1}{\partial s_{10}} \dot{s}_{10} + \frac{\partial Q_1}{\partial t}$$

Darcy : $\dot{Q}_1 = 2\pi T_1 r \frac{ds_1}{dr} = -2\pi T_1 \frac{s_{10}}{u_{10}}$ d'où : $\int_a^R \dot{Q}_1 \delta Q_1 dr = -2\pi T_1 \frac{s_{10}}{u_{10}} \int_a^R \delta Q_1 dr$ ou :

$$\dot{u}_{10} \int_0^{u_{10}} \left(\frac{\partial Q_1}{\partial u_{10}}\right)^2 du_1 + \dot{s}_{10} \int_0^{u_{10}} \frac{\partial Q_1}{\partial s_{10}} \frac{\partial Q_1}{\partial u_{10}} du_1 + \int_0^{u_{10}} \frac{\partial Q_1}{\partial u_{10}} \frac{\partial Q_1}{\partial t} du_1 = -2\pi T_1 \frac{s_{10}}{u_{10}} \int_0^{u_{10}} \frac{\partial Q_1}{\partial u_{10}} du_1 \quad (')$$

or $$\dot{u}_{10} \int_0^{u_{10}} \left(\frac{\partial Q_1}{\partial u_{10}}\right)^2 du_1 = \dot{u}_{10} (-2\pi S_2)^2 s_{10}^2 (1-e^{-\varepsilon t})^2 \left(\frac{a^2}{4u_{10}^2 e^{-2u_{10}}}\right)^2 \underbrace{\int_0^{u_{10}} \left[u_{10} \frac{\partial \varphi_1}{\partial u_1} + (2u_{10}-1)\varphi_1\right]^2 du_1}_{I_1}$$

$$\dot{s}_{10} \int_0^{u_{10}} \frac{\partial Q_1}{\partial s_{10}} \frac{\partial Q_1}{\partial u_{10}} du_1 = \dot{s}_{10} (-2\pi S_2)^2 s_{10} (1-e^{-\varepsilon t})^2 \left(\frac{a^2}{4u_{10}^2 e^{-2u_{10}}}\right)^2 u_{10} \underbrace{\int_0^{u_1} \varphi_1 \left[u_{10} \frac{\partial \varphi_1}{\partial u_1} + (2u_{10}-1)\varphi_1\right] du_1}_{I_2}$$

$$\int_0^{u_{10}} \frac{\partial Q_1}{\partial u_1} \frac{\partial Q_1}{\partial t} du_1 = (-2\pi S_2)^2 s_{10}^2 \left(\frac{a^2}{4u_{10}^2 e^{-2u_{10}}}\right)^2 u_{10} \varepsilon e^{-\varepsilon t} (1-e^{-\varepsilon t}) \underbrace{\int_0^{u_1} \varphi_1 \left[u_{10} \frac{\partial \varphi_1}{\partial u_1} + (2u_{10}-1)\varphi_1\right] du_1}_{I_2}$$

$$-2\pi T_1 \frac{s_{10}}{u_{10}} \int_0^{u_{10}} \frac{\partial Q_1}{\partial u_{10}} du_1 = -2\pi T_1 \frac{s_{10}}{u_{10}} (-2\pi S_2) s_{10} (1-e^{-\varepsilon t}) \frac{a^2}{4u_{10}^2 e^{-2u_{10}}} \underbrace{\int_0^{u_{10}} \left[u_{10} \frac{\partial \varphi_1}{\partial u_1} + (2u_{10}-1)\varphi_1\right] du_1}_{I_4}$$

et (') devient, toutes réductions faites :

$$(1-e^{-\varepsilon t}) I_1 \frac{\dot{u}_{10}}{u_{10}} + (1-e^{-\varepsilon t}) I_2 \frac{\dot{s}_{10}}{s_{10}} + \varepsilon e^{-\varepsilon t} I_2 = \frac{4T_1}{S_2 a^2} e^{-2u_{10}} I_4 \quad (^2)$$

La contrainte $\dot{Q}_{10}$ = constante s'écrit $\dot{Q}_{10} t = Q_{10}$ ou

$$\frac{\dot{s}_{10}}{s_{10}} = \frac{1}{t} - \frac{\psi_{10}}{\varphi_{10}} \frac{\dot{u}_{10}}{u_{10}} - \frac{\varepsilon e^{-\varepsilon t}}{1-e^{-\varepsilon t}} \quad \text{et } (^2) \text{ devient :}$$

$$\frac{I_1 \varphi_{10} - I_2 \psi_{10}}{\varphi_{10}} \frac{\dot{u}_{10}}{u_{10}} + \frac{I_2}{t} = \frac{4T_1}{S_2 a^2} e^{-2u_{10}} \frac{I_4}{1-e^{-\varepsilon t}} \quad (^3)$$

Soit $\frac{4T_1 t}{S_2 a^2} = \alpha$, $\frac{I_1 \varphi_{10} - I_2 \psi_{10}}{u_{10}} = F_{10}$ et $\varepsilon t = \eta\alpha$ (3) devient :

$$\boxed{\frac{du_{10}}{d\alpha} = \left[\frac{I_4 e^{-2u_{10}}\alpha}{1-e^{-\eta\alpha}} - I_2\right]\frac{\varphi_{10}}{F_{10}\alpha}}$$

Pour $\alpha \to \infty$, on retrouve la formule valable en milieu homogène (1.2.2)

Pour $\eta\alpha$ petit et u_{10} grand, on trouve $u_{10} = (\alpha\eta)^{-4/3}$ soit, pour $\alpha\eta = 0.01$

$u_{10} = 464,1588$.

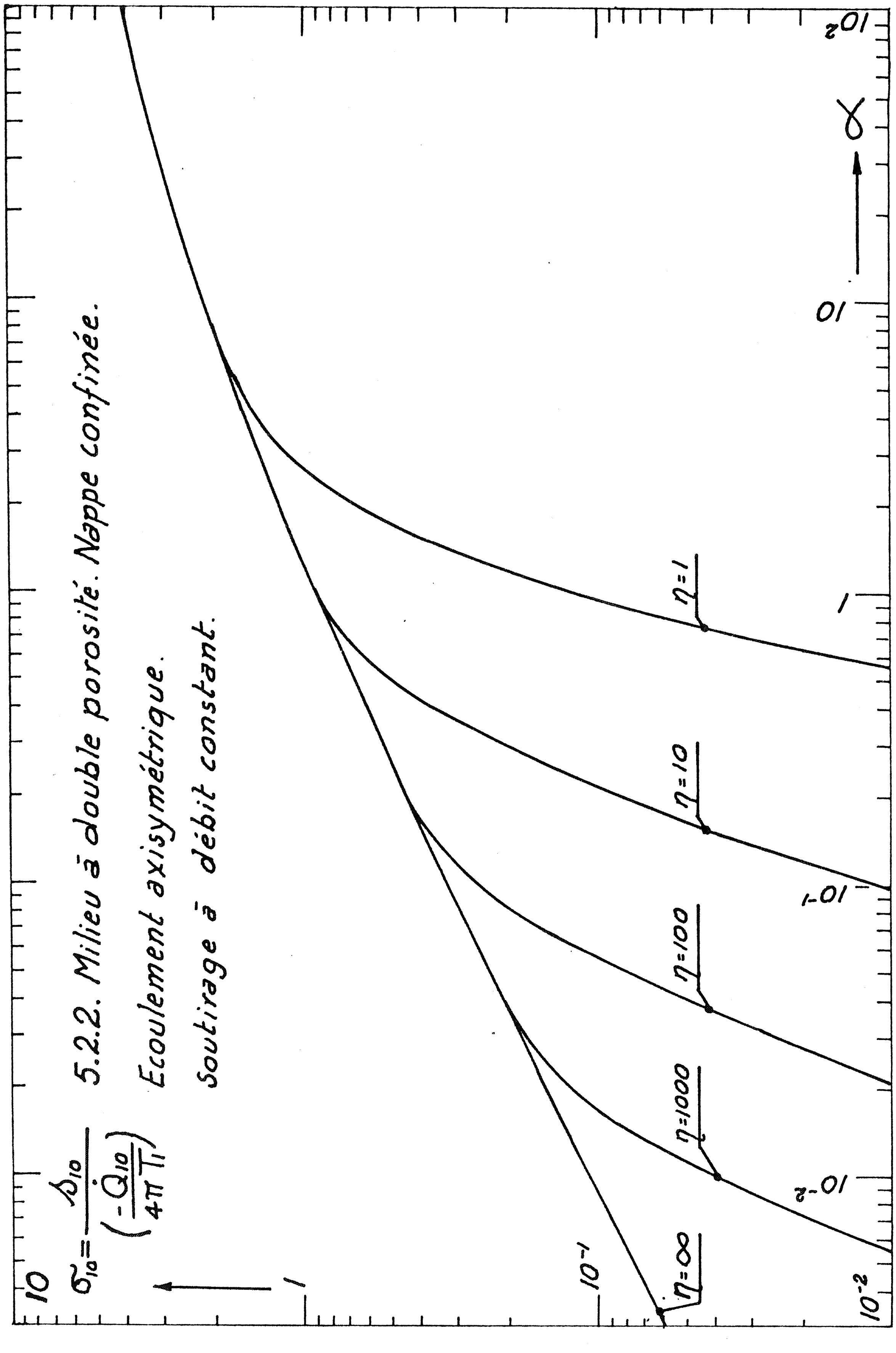
$\dot{Q}_{10}\, t = Q_{10}$ devient $\dot{Q}_{10}\, t = (-2\pi S_2)\, s_{10}(1-e^{-\varepsilon t})\frac{a^2}{4u_{10}^2 e^{-2u_{10}}}\, u_{10}\, \varphi_{10}$ ou :

$$\boxed{\frac{s_{10}}{-\frac{\dot{Q}_{10}}{4\pi T_1}} = \frac{2u_{10}\, e^{-2u_{10}}}{\varphi_{10}(1-e^{-\eta\alpha})}\,\alpha}$$

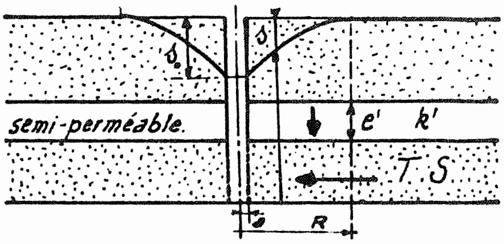
Pour $\eta \to \infty$, on retrouve la formule du problème 1.2.2.

$$\sigma_{10} = \frac{s_{10}}{-\dot{Q}_{10}/4\pi T_1}$$

α	$\eta=1$		$\eta=10$		$\eta=100$		$\eta=1000$		$\eta=10^5$	
	u_{10}	σ_{10}	u_{10}	σ_{10}	u_{10}	σ_{10}	u_{10}	σ_{10}	u_{10}	σ_{10}
10^{-7}									464.1	0
2									184.9	0
3									108.2	0
5									55.42	~ 0
7									35.85	~ 0
10^{-6}									22.77	~ 0
2									9.803	~ 0
3									6.233	$1\ 10^{-9}$
5									3.744	$5.4\ 10^{-8}$
7									2.762	$3.1\ 10^{-7}$
10^{-5}							464.1	0	2.024	$7.2\ 10^{-6}$
2							184.9	0	1.085	$8.9\ 10^{-6}$
3							108.2	0	0.728	$2.5\ 10^{-5}$
5							55.42	~ 0	0.4229	$8.7\ 10^{-5}$
7							35.85	~ 0	0.2888	$1.98\ 10^{-4}$
10^{-4}					464.1	0	22.77	~ 0	0.1898	$4.62\ 10^{-4}$
2					184.9	0	9.803	~ 0	0.08203	$2.30\ 10^{-3}$
3					108.2	0	6.233	~ 0	0.05102	0.00568
5					55.42	~ 0	3.7438	~ 0	0.03186	0.01535
7					35.85	~ 0	2.763	~ 0	0.027238	0.02523
10^{-3}			464.1	0	22.77	~ 0	2.0248	~ 0	0.02867	0.0342
2			184.9	0	9.803	~ 0	1.0852	~ 0	etc.	etc.
3			108.2	0	6.233	~ 0	0.7293	0.00250		
5			55.42	~ 0	3.744	$5.3\ 10^{-5}$	0.4256	0.00872		
7			35.85	~ 0	2.763	$3.1\ 10^{-4}$	0.2953	0.01929		
10^{-2}	464.1	0	22.77	~ 0	2.025	0.001223	0.2052	0.04229		
2	184.9	0	9.803	$7.\ 10^{-9}$	1.087	0.008949	0.14258	0.1272		
3	108.2	~ 0	6.233	$5.55\ 10^{-6}$	0.7351	0.02469	0.15218	0.1776		
5	55.42	~ 0	3.744	$5.35\ 10^{-4}$	0.4487	0.08126	0.1858	0.2368		
7	35.85	~ 0	2.763	$3.14\ 10^{-3}$	0.3464	0.15836	0.2157	0.2795		
10^{-1}	22.77	$7.9\ 10^{-19}$	2.026	0.0122	0.3050	0.2646	0.2530	0.3314		
2	9.803	$6.6\ 10^{-8}$	1.103	0.0867	0.3493	0.4473	0.3431	0.4575		
3	6.233	$5.5\ 10^{-5}$	0.7884	0.2199	0.4093	0.5472	0.4076	0.5501		
5	3.744	0.0053	0.6144	0.5204	0.5032	0.6898	0.5029	0.6904		
7	2.765	0.0312	0.6149	0.7235	0.5748	0.7988	0.5747	0.7990		
1	2.040	0.1193	0.6712	0.9033	0.6590	0.9294	0.65899	0.9295		
2	1.243	0.6743	0.8483	1.2299	0.8472	1.2326	0.8472	1.2326		
3	1.110	1.1707	0.9724	1.4415	0.9721	1.4422	0.9721	1.4422		
5	1.171	1.6678	1.1443	1.7408	1.1443	1.74099	1.1443	1.74099		
7	1.274	1.9327	1.2661	1.9587	1.2661	1.9587				
10	1.403	2.1996	1.4019	2.2071	1.4019	2.2071				
20	1.683	2.7365	1.683	2.7370						
30			1.856	3.0723						
50			2.082	3.5166						
70										
100										


5.2.2. Milieu à double porosité. Nappe confinée.
Ecoulement axisymétrique.
Soutirage à débit constant.
$\sigma_{10} = \dfrac{s_{10}}{\left(\dfrac{-\dot{Q}_{10}}{4\pi T_1}\right)}$
10
1
10^{-1}
10^{-2}
γ
10^{2}
10
1
10^{-1}
10^{-2}
$\eta = \infty$
$\eta = 1000$
$\eta = 100$
$\eta = 10$
$\eta = 1$

6.1. Nappe confinée. Ecoulement axisymétrique. Soutirage à niveau constant. Prise en compte d'une drainance verticale (Leaky confined aquifers)



Formule analytique. Hantush. 1959.

$$\frac{\dot{Q}_0}{(-2\pi T)s_0} = G\left(\alpha, \frac{a}{B}\right) \qquad \alpha = \frac{Tt}{Sa^2} \qquad B = \sqrt{\frac{k'}{e'T}}$$

$$G\left(\alpha, \frac{a}{B}\right) = \frac{a}{B}\frac{K_1\left(\frac{a}{B}\right)}{K_0\left(\frac{a}{B}\right)} + \frac{4}{\pi^2} e^{-\alpha\left(\frac{a}{B}\right)^2} \int_0^\infty \frac{u e^{-\alpha u^2}\, du}{\left[J_0^2(u) + Y_0^2(u)\right]\left(u^2 + \frac{a^2}{B^2}\right)}$$

Continuité : $\dfrac{d\dot{Q}}{dr} = 2\pi r S \dot{s} + 2\pi r \dfrac{s}{c} = 2\pi r S\left(\dot{s} + \dfrac{s}{cS}\right) = 2\pi r S \dfrac{d}{dt}\left(s + \dfrac{st}{cS}\right)$

d'où $\dfrac{dQ}{dr} = 2\pi r S s\left(1 + \dfrac{t}{cS}\right)$ avec $c = \dfrac{e'}{k'}$

d'où $Q = -2\pi S\left(1 + \dfrac{t}{cS}\right)\displaystyle\int_r^R r s\, dr$ $\dfrac{1}{c}$ = drainance.

$$Q = -2\pi S\left(1+\frac{t}{cS}\right)\frac{s_0 a^2}{u_0 e^{-2u_0}}\left(-\frac{1}{2}u e^{-2u} - \frac{1}{4}e^{-2u} + \frac{1}{4}\right) = -\left(1+\frac{t}{cS}\right)\frac{\pi S a^2 s_0 u_0}{2u_0^2 e^{-2u_0}}\varphi$$

$$\frac{\partial Q}{\partial u_0} = -\left(1+\frac{t}{cS}\right)\left(\frac{\pi S a^2}{2u_0^2 e^{-2u_0}}\right)s_0\left[u_0\frac{\partial\varphi}{\partial u} + (2u_0-1)\varphi\right]$$

$$\frac{\partial Q}{\partial t} = -\frac{1}{cS}\frac{\pi S a^2}{2u_0^2 e^{-2u_0}} s_0 u_0 \varphi \quad \text{d'où} \quad \frac{\partial Q_0}{\partial u_0} = -\left(1+\frac{t}{cS}\right)\frac{\pi S a^2}{2u_0^2 e^{-2u_0}} s_0 \varphi_0$$

$$\text{et} \quad \frac{\partial Q_0}{\partial t} = -\frac{1}{cS}\left(\frac{\pi S a^2}{2u_0^2 e^{-2u_0}}\right) s_0 u_0 \varphi_0$$

$$\dot{Q} = \frac{\partial Q}{\partial u_0}\dot{u}_0 + \frac{\partial Q}{\partial t} = -\left(\frac{\pi S a^2}{2u_0^2 e^{-2u_0}}\right)\left\{\left(1+\frac{t}{cS}\right)s_0\left[u_0\frac{\partial\varphi}{\partial u} + (2u_0-1)\varphi\right]\dot{u}_0 + \frac{1}{cS}s_0 u_0 \varphi\right\}$$

Darcy : $\displaystyle\int_0^{u_0} \dot{Q}\frac{\partial Q}{\partial u_0}du = -2\pi T\frac{s_0}{u_0}\int_0^{u_0}\frac{\partial Q}{\partial u_0}du$ devient :

$$\left(\frac{\pi S a^2}{2u_0^2 e^{-2u_0}}\right)^2\left(1+\frac{t}{cS}\right)s_0\left\{s_0\left(1+\frac{t}{cS}\right)\dot{u}_0\underbrace{\int_0^{u_0}\left[u_0\frac{\partial\varphi}{\partial u} + (2u_0-1)\varphi\right]^2 du}_{I_1} + \frac{s_0 u_0}{cS}\underbrace{\int_0^{u_0}\varphi\left[u_0\frac{\partial\varphi}{\partial u} + (2u_0-1)\varphi\right]du}_{I_2}\right.$$

$$= 2\pi T\frac{s_0}{u_0}\left(1+\frac{t}{cS}\right)\left(\frac{\pi S a^2}{2u_0^2 e^{-2u_0}}\right)s_0\underbrace{\int_0^{u_0}\left[u_0\frac{\partial\varphi}{\partial u} + (2u_0-1)\varphi\right]du}_{I_4}$$

ou, toutes réductions faites :

$$\boxed{\left(1+\frac{t}{cS}\right)\frac{I_1}{I_4}\frac{\dot{u}_0}{u_0}+\frac{I_2}{cSI_4}=\frac{4Te^{-2u_0}}{Sa^2}}$$

Pour $c=\infty$ ($e'\infty$ ou k' nul) (drainance nulle) on retrouve la formule du problème 1.2.1

Soit $\frac{Tt}{Sa^2}=\alpha$, il vient, en posant $B^2=\frac{Te'}{k'}=Tc$.

$$\frac{du_0}{d\alpha}=\frac{I_4 u_0}{I_1\left(1+\frac{a^2}{B^2}\alpha\right)}\left[4e^{-2u_0}-\frac{I_2}{I_4}\frac{a^2}{B^2}\right]$$ qu'on peut écrire :

$$\frac{du_0^2}{4-\frac{9}{10}\frac{a^2}{B^2}u_0^2}=\frac{5}{4}\frac{d\alpha}{1+\frac{a^2}{B^2}\alpha}$$ qui devient, pour α et u_0 petits :

$\left(1+\frac{9}{40}\frac{a^2}{B^2}u_0^2\right)du_0^2=5\left(1-\frac{a^2}{B^2}\alpha\right)d\alpha$ dont la solution, pour α et u_0 petits est $u_0^2=5\alpha$

$\dot{Q}_0=\frac{\partial Q_0}{\partial u_0}\dot{u}_0+\frac{\partial Q_0}{\partial t}$ devient : $\dot{Q}_0=\frac{-\pi Sa^2}{2u_0^2e^{-2u_0}}s_0u_0\left[\left(1+\frac{t}{cS}\right)\varphi_0\frac{\dot{u}_0}{u_0}+\frac{\varphi_0}{cS}\right]$

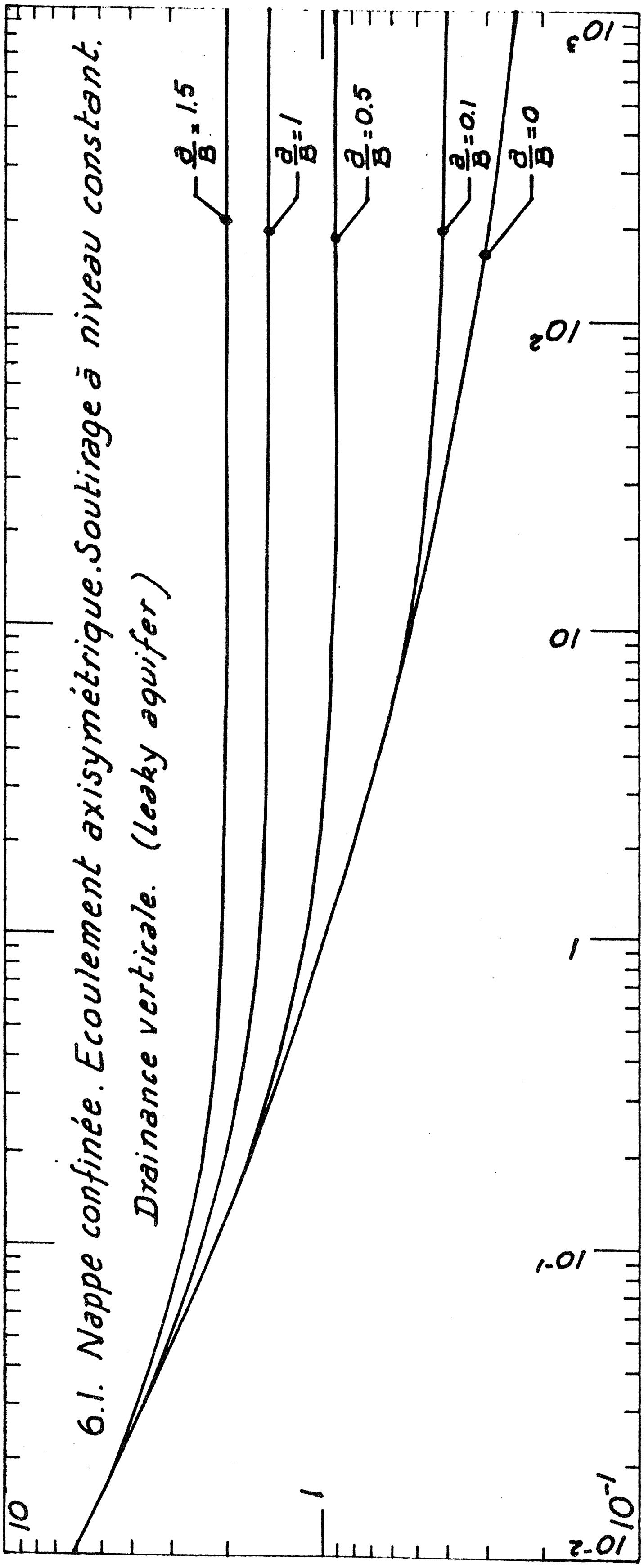
et puisque $\frac{\dot{u}_0}{u_0}=\frac{\frac{4Te^{-2u_0}}{Sa^2}-\frac{I_2}{cSI_4}}{\left(1+\frac{t}{cS}\right)\frac{I_1}{I_4}}$ il vient $\frac{\dot{Q}_0}{(-2\pi T)s_0}=\frac{\varphi_0 I_4}{u_0 I_1}+\frac{a^2F}{4e^{-2u_0}I_1Tc}$

ou, enfin $$\boxed{\frac{\dot{Q}_0}{(-2\pi T)s_0}=\frac{\varphi_0 I_4}{u_0 I_1}+\frac{F}{4e^{-2u_0}I_1}\frac{a^2}{B^2}}$$

Pour $\frac{a^2}{B^2}=0$ ($B=\infty$), on retrouve la formule obtenue en 1.2.1.

	$a/B=0.1$		$a/B=0.5$		$a/B=1$		$a/B=1.5$	
α	u_0	$-\dot{Q}_0/2\pi Ts_0$	u_0	$-\dot{Q}_0/2\pi Ts_0$	u_0	$-\dot{Q}_0/2\pi Ts_0$	u_0	$-\dot{Q}_0/2\pi Ts_0$
10^{-4}	0.02236	59.050	0.02236	59.050	0.02236	59.048	0.02236	54.135
2	0.03126	39.624	0.03131	39.869	0.03127	39.668	0.03123	38.879
3	0.03811	32.612	0.03814	32.732	0.03813	32.514	0.03808	32.538
5	0.04890	25.462	0.04893	25.443	0.04891	25.455	0.04886	25.453
7	0.05760	21.594	0.05762	21.581	0.05760	21.594	0.05754	21.642
10^{-3}	0.06847	18.155	0.06849	18.145	0.06846	18.168	0.06838	18.209
2	0.09556	12.978	0.09555	12.984	0.09548	13.010	0.09534	13.058
3	0.11588	10.684	0.11585	10.693	0.11572	10.726	0.11549	10.781
5	0.14732	8.3833	0.14725	8.396	0.14697	8.4378	0.14651	8.5077
7	0.17220	7.1581	0.17207	7.173	0.17163	7.2229	0.17090	7.3050
10^{-2}	0.20274	6.0653	0.20252	6.084	0.20179	6.1429	0.20060	6.2404
2	0.27639	4.4242	0.27578	4.450	0.27389	4.5338	0.27083	4.6699
3	0.32953	3.6960	0.32847	3.728	0.32521	3.8299	0.31999	3.9946
5	0.40854	2.9642	0.40643	3.006	0.40006	3.1361	0.39009	3.3438
7	0.46853	2.5738	0.46526	2.624	0.45547	2.7760	0.44050	3.0162
10^{-1}	0.53944	2.2247	0.53425	2.284	0.51903	2.4641	0.49648	2.7421
2	0.69959	1.6980	0.68732	1.782	0.65312	2.0263	0.60679	2.3830
3	0.80710	1.4626	0.78718	1.565	0.73426	1.8531	0.66758	2.2548
5	0.95665	1.2241	0.92087	1.355	0.83342	1.7019	0.73499	2.1535
7	1.06328	1.0957	1.01154	1.249	0.89380	1.6346	0.77213	2.1127
1	1.18278	0.9799	1.10754	1.160	0.95129	1.5845	0.80467	2.0846

	a/B = 0.1		a/B = 0.5		a/B = 1		a/B = 1.5	
α	U_0	$-\dot{Q}_0/2\pi T\delta_0$	U_0	$-\dot{Q}_0/2\pi T\delta_0$	U_0	$-\dot{Q}_0/2\pi T\delta_0$	U_0	$-\dot{Q}_0/2\pi T\delta_0$
1	1.18278	0.9799	1.1075	1.1600	0.9513	1.5845	0.8046	2.0846
2	1.4315	0.8029	1.2842	1.0429	1.0395	1.5303	0.8492	2.0563
3	1.5850	0.7226	1.3754	1.0010	1.0764	1.5146	0.8659	2.0486
5	1.7840	0.6406	1.4719	0.9680	1.1096	1.5039	0.8800	2.0432
7	1.9168	0.5964	1.5232	0.9547	1.1250	1.4999	0.8861	2.0411
10	2.0574	0.5568	1.5672	0.9455	1.1369	1.4972	0.8908	2.0397
2	2.3230	0.4982	1.6259	0.9363	1.1512		0.8962	
3	2.4686	0.4735	1.6476	0.9337	1.1561		0.8980	
5	2.6350	0.4508	1.6658	0.9319	1.1599		0.8994	
7	2.7314	0.4402	1.6737	0.9313	1.1615		0.9000	
10^2	2.8202	0.4320	1.6797	0.9308	1.1628	1.4926	0.9005	2.03711
2	2.9521	0.4228	1.6868	0.9303	1.1641			
3	3.0060	0.4201	1.6891	0.9301	1.1646			
5	3.0540	0.4180	1.6909	0.9300	1.1649		0.9012	2.03692
7	3.0761	0.4173	1.6917	0.92998	1.1651			
10^3	3.0933	0.4168	1.6923	0.92994	1.1652	1.4923		
2	3.1141	0.4162	1.6930		1.1653			
3	3.1212	0.4161	1.6932		1.16540			
5	3.1268	0.41597	1.6933		1.16543			
7	3.1293	0.41592	1.6934		1.16544			
10^4	3.1311	0.41588	1.69351	0.92987	1.16545	1.492272		
2	3.1332	0.41584	1.69357		1.16546			
3	3.1339	0.41583	1.69359		1.165472			
5	3.1344	0.41582	1.693609		1.165474			
7	3.1346	0.4158157	1.693616		1.1654760			
10^5	3.1348	0.4158124	1.693621	0.929867	1.1654769	1.492270		
2	3.1350	0.415808	1.693626					
3	3.13511	0.415807	1.693628					
5	3.13516	0.4158064	1.693629					
7	3.13518	0.4158060	1.6936303					
10^6	3.13520	0.4158057	1.6936308	0.929866	1.1654786	1.492269		
2	3.135220	0.4158053						
3	3.135226	"						
5	3.135231	"						
7	3.135233	"						
10^7	3.135235	0.415805						



6.1. Nappe confinée. Ecoulement axisymétrique. Soutirage à niveau constant. Drainance verticale. (Leaky aquifer)

6.2. Nappe confinée. Ecoulement axisymétrique. Soutirage à débit constant. Drainance verticale. (Leaky aquifer).

Formule analytique. (Hantush and Jacob, 1955).

$$\frac{s}{\left(\frac{-Q_0}{4\pi T}\right)} = 2K_0(2v) - \int_{\frac{v^2}{u}}^{\infty} \frac{1}{y} e^{-\frac{y+v^2}{y}} dy \quad \text{avec} \quad v = \frac{r}{2}\sqrt{\frac{k'}{e'T}} \qquad u = \frac{Sr^2}{4Tt}$$

Continuité : $\frac{d\dot{Q}}{dr} = 2\pi r S \frac{d}{dt}\left(s + \frac{st}{cS}\right)$ avec $\frac{1}{c} = \frac{k'}{e'}$

d'où $\frac{dQ}{dr} = 2\pi r S\left(1 + \frac{t}{cS}\right)s$.

$$Q = -2\pi S\int_r^R rs\left(1+\frac{t}{cS}\right)dr = -2\pi S\left(1+\frac{t}{cS}\right)\int_r^R rs\,dr = -\left(1+\frac{t}{cS}\right)\frac{\pi S a^2 s_0 u_0}{2u_0^2 e^{-2u_0}}\varphi$$

$$\frac{\partial Q}{\partial u_0} = -\left(1+\frac{t}{cS}\right)\frac{\pi S a^2 s_0}{2u_0^2 e^{-2u_0}}\left[u_0\frac{\partial\varphi}{\partial u} + (2u_0-1)\varphi\right]$$

$$\frac{\partial Q}{\partial s_0} = -\left(1+\frac{t}{cS}\right)\frac{\pi S a^2 u_0}{2u_0^2 e^{-2u_0}}\varphi$$

$$\frac{\partial Q}{\partial t} = -\frac{1}{cS}\frac{\pi S a^2 u_0 s_0}{2u_0^2 e^{-2u_0}}\varphi \quad \text{d'où}$$

$$\dot{Q} = -\left(1+\frac{t}{cS}\right)\frac{\pi S a^2 s_0}{2u_0^2 e^{-2u_0}}\left[u_0\frac{\partial\varphi}{\partial u} + (2u_0-1)\varphi\right]\dot{u}_0 - \left(1+\frac{t}{cS}\right)\frac{\pi S a^2 u_0}{2u_0^2 e^{-2u_0}}\varphi\dot{s}_0 - \frac{1}{cS}\frac{\pi S a^2 u_0 s_0}{2u_0^2 e^{-2u_0}}\varphi$$

Darcy : $\dot{Q} = 2\pi r T\frac{ds}{dr}$ ou $\int_a^R \dot{Q}\,\delta Q\,\frac{dr}{r} = 2\pi T\int_a^R \frac{ds}{dr}\,\delta Q\,dr$ avec

$$\dot{Q} = \frac{\partial Q}{\partial u_0}\dot{u}_0 + \frac{\partial Q}{\partial s_0}\dot{s}_0 + \frac{\partial Q}{\partial t} \qquad \delta Q = \frac{\partial Q}{\partial u_0}\delta u_0 + \frac{\partial Q}{\partial s_0}\delta s_0$$

d'où $\int_0^{u_0} \dot{Q}\frac{\partial Q}{\partial u_0}du = -2\pi T\frac{s_0}{u_0}\int_0^{u_0}\frac{\partial Q}{\partial u_0}du$ $(')$ or:

$$\dot{Q}\frac{\partial Q}{\partial u_0} = -\frac{\pi S a^2 u_0 s_0}{2u_0^2 e^{-2u_0}}\left[\left(1+\frac{t}{cS}\right)\left(u_0\frac{\partial\varphi}{\partial u} + (2u_0-1)\varphi\right)\frac{\dot{u}_0}{u_0} + \left(1+\frac{t}{cS}\right)\varphi\frac{\dot{s}_0}{s_0} + \frac{\varphi}{cS}\right] \times \left\{-\left(1+\frac{t}{cS}\right)\frac{\pi S a^2 s_0}{2u_0^2 e^{-2u_0}}\left[u_0\frac{\partial\varphi}{\partial u} + (2u_0-1)\varphi\right]\right\}$$

ou $\dot{Q}\frac{\partial Q}{\partial u_0} = \left(\frac{-\pi S a^2 s_0}{2u_0^2 e^{-2u_0}}\right)^2 u_0\left(1+\frac{t}{cS}\right)\left\{\left(1+\frac{t}{cS}\right)\left[u_0\frac{\partial\varphi}{\partial u} + (2u_0-1)\varphi\right]\frac{\dot{u}_0}{u_0} + \left(1+\frac{t}{cS}\right)\varphi\frac{\dot{s}_0}{s_0} + \frac{\varphi}{cS}\right\}\left[u_0\frac{\partial\varphi}{\partial u} + (2u_0-1)\varphi\right]$

et (') devient :

et (') devient :

$$\left(\frac{\pi S a^2 s_0}{2u_0^2 e^{-2u_0}}\right)^2 u_0\left(1+\frac{t}{cS}\right)\left\{\left(1+\frac{t}{cS}\right)\frac{\dot u_0}{u_0}\int_0^{u_0}\left[u_0\frac{\partial\varphi}{\partial u}+(2u_0-1)\varphi\right]^2 du\right.$$

$$+\left(1+\frac{t}{cS}\right)\frac{\dot s_0}{s_0}\int_0^{u_0}\varphi\left[u_0\frac{\partial\varphi}{\partial u}+(2u_0-1)\varphi\right]du$$

$$\left.+\frac{1}{cS}\int_0^{u_0}\varphi\left[u_0\frac{\partial\varphi}{\partial u}+(2u_0-1)\varphi\right]du\right\}$$

$$=2\pi T\frac{s_0}{u_0}\left(1+\frac{t}{cS}\right)\left(\frac{\pi S a^2 s_0}{2u_0^2 e^{-2u_0}}\right)\int_0^{u_0}\left[u_0\frac{\partial\varphi}{\partial u}+(2u_0-1)\varphi\right]$$ ou, toutes réductions faites :

$$\left(1+\frac{t}{cS}\right)I_1\frac{\dot u_0}{u_0}+\left(1+\frac{t}{cS}\right)I_2\frac{\dot s_0}{s_0}+\frac{1}{cS}I_2=\frac{4T}{Sa^2}e^{-2u_0}I_4 \qquad (^2)$$

$\dot Q_0 t = Q_0$ devient :

$$\left(1+\frac{t}{cS}\right)\frac{\pi S a^2 s_0}{2u_0^2e^{-2u_0}}\psi_0\dot u_0+\left(1+\frac{t}{cS}\right)\frac{\pi S a^2 u_0\varphi_0\dot s_0}{2u_0^2e^{-2u_0}}+\frac{1}{cS}\frac{\pi S a^2 u_0 s_0\varphi_0}{2u_0^2e^{-2u_0}}=\frac{1+\frac{t}{cS}}{t}\cdot\frac{\pi S a^2 s_0 u_0\varphi_0}{2u_0^2e^{-2u_0}}$$

ou $\dfrac{\dot s_0}{s_0}=\dfrac{cS}{t(cS+t)}-\dfrac{\psi_0}{\varphi_0}\dfrac{\dot u_0}{u_0}$ et $(^2)$ devient :

$$\frac{\varphi_0 I_1-\psi_0 I_2}{\varphi_0 I_4}\frac{\dot u_0}{u_0}+\frac{I_2}{tI_4}=\frac{4T}{Sa^2}e^{-2u_0}\frac{cS}{cS+t}$$. Soit $\alpha=\dfrac{4Tt}{Sa^2}$, il vient :

$$\frac{F}{\varphi_0 I_4}\frac{du_0}{d\alpha}=\frac{4}{4+\frac{a^2}{Tc}\alpha}-\frac{I_2}{I_4\alpha}$$ Posons $B^2=Tc$ il vient :

$$\boxed{\frac{du_0}{d\alpha}=\frac{\varphi_0 I_4}{F}\left[\frac{4e^{-2u_0}}{4+\frac{a^2}{B^2}\alpha}-\frac{I_2}{I_4\alpha}\right]}$$

Pour $\frac{a^2}{B^2}=0$ $(B=\infty)$, on retrouve la formule du problème 1.2.2.

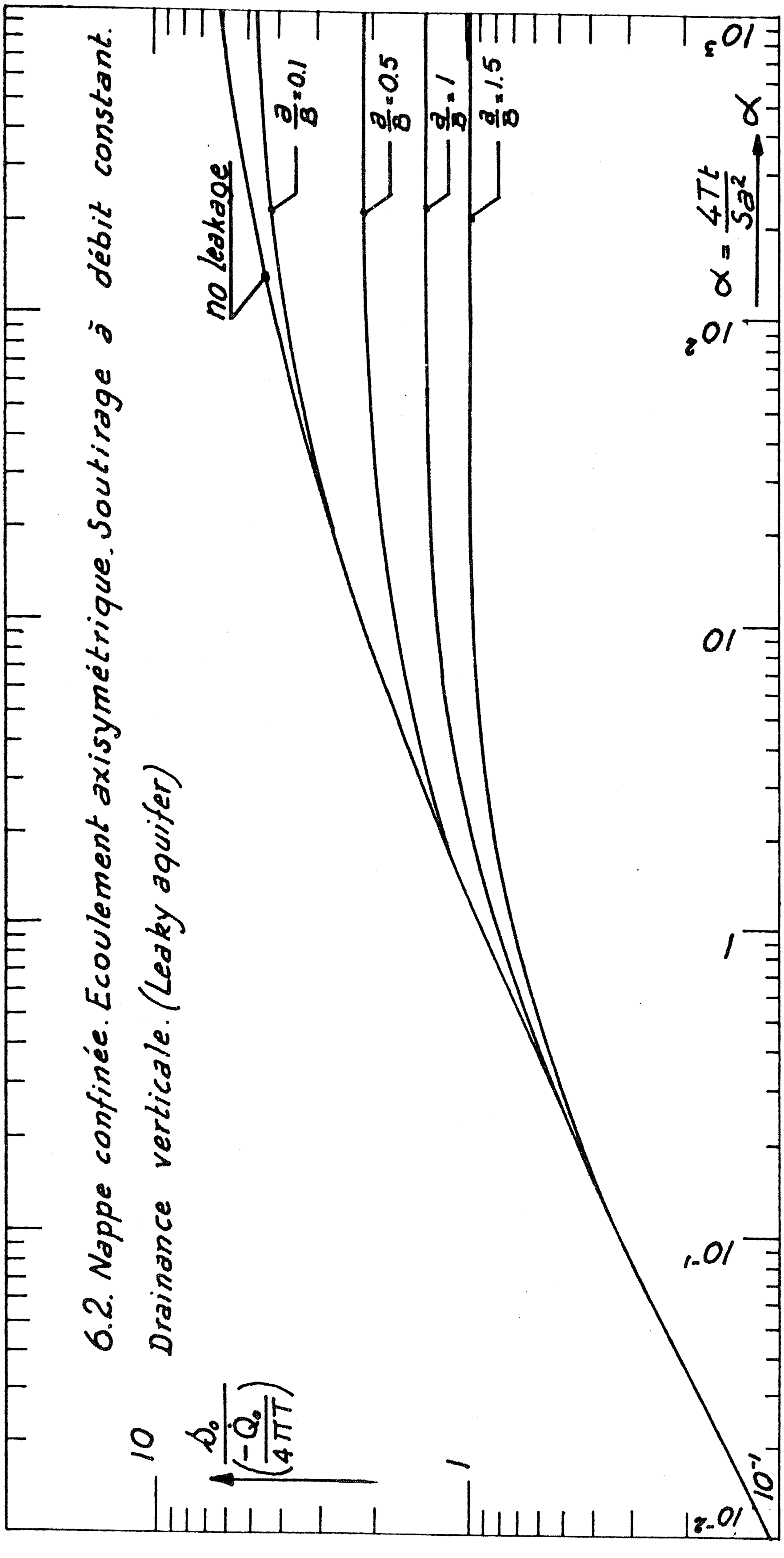
Pour u_0 petit $\dfrac{du_0}{d\alpha}\approx\dfrac{2u_0^2\cdot\frac{4}{3}u_0^3}{\frac{28}{15}u_0^6}\left[\dfrac{4}{4+\frac{a^2}{B^2}\alpha}-\dfrac{\frac{6}{5}u_0^5}{\frac{4}{3}u_0^3\alpha}\right]$ dont la solution est $u_0=\sqrt{\frac{4}{5}\alpha}$

$\dot Q_0 t = Q_0$ donne : $\dot Q_0 t=-\left(1+\dfrac{t}{cS}\right)\dfrac{\pi S a^2 s_0 u_0}{2u_0^2e^{-2u_0}}\varphi_0$ ou

$$\boxed{\frac{s_0}{\left(-\frac{Q_0}{4\pi T}\right)}=\frac{2u_0e^{-2u_0}}{\varphi_0}\alpha\frac{1}{1+\frac{a^2}{B^2}\frac{\alpha}{4}}}$$

A une distance r du puits, on a : $s=\dfrac{s_0}{u_0}u=\dfrac{s_0}{u_0}\left(u_0-\log\dfrac{r}{a}\right)$ et on connaît u_0 en fonction de α.

α	$a^2/B^2 = 0.01$		$a^2/B^2 = 0.25$		$a^2/B^2 = 1$		$a^2/B^2 = 2.25$	
	U_0	σ_0	U_0	σ_0	U_0	σ_0	U_0	σ_0
10^{-3}	0.02828	0.03469	0.02828	0.03469	0.02828	0.03468	0.02828	0.03467
2	0.03944	0.04937	0.03936	0.04947	0.03961	0.04913	0.03956	0.04916
3	0.04800	0.06051	0.04788	0.06065	0.04810	0.06034	0.04796	0.06046
5	0.06157	0.07790	0.06152	0.07794	0.06155	0.07783	0.06152	0.07775
7	0.07252	0.09190	0.07249	0.09191	0.07246	0.09182	0.07241	0.09169
10^{-2}	0.08616	0.10949	0.08613	0.10946	0.08607	0.10933	0.08597	0.10913
2	0.12003	0.15354	0.1198	0.1534	0.11981	0.15309	0.11954	0.15252
3	0.14537	0.18684	0.1452	0.1866	0.14497	0.18602	0.14448	0.18500
5	0.18442	0.23879	0.1842	0.2383	0.18360	0.23706	0.18259	0.23493
7	0.21521	0.28027	0.2148	0.2795	0.21389	0.27745	0.21226	0.27401
10^{-1}	0.25285	0.33163	0.2523	0.3304	0.25069	0.32692	0.24804	0.32125
2	0.34302	0.45744	0.3416	0.4543	0.33744	0.44492	0.33081	0.43045
3	0.40756	0.54992	0.4051	0.5443	0.39801	0.52805	0.38697	0.50374
5	0.50274	0.68995	0.4980	0.6786	0.48432	0.64666	0.46410	0.60194
7	0.57442	0.79823	0.5671	0.7804	0.54649	0.73147	0.51718	0.66692
1	0.65851	0.92826	0.6471	0.8995	0.61576	0.82449	0.57358	0.73265
2	0.84607	1.22946	0.8198	1.1596	0.75381	1.0009	0.67639	0.84017
3	0.97024	1.4367	0.9284	1.3226	0.83140	1.09162	0.72821	0.88637
5	1.14081	1.7305	1.0679	1.5270	0.91963	1.18379	0.78178	0.92691
7	1.26093	1.9428	1.1581	1.6533	0.96972	1.22944	0.80966	0.944608
10	1.39412	2.18243	1.2493	1.7738	1.01486	1.26556	0.83325	0.95750
2	1.66624	2.68109	1.4042	1.9540	1.07958	1.30746	0.86453	0.97132
3	1.83047	2.9840	1.4769	2.0242	1.10518	1.32030	0.87608	0.97537
5	2.03827	3.3636	1.5485	2.0815	1.12762	1.32955	0.88583	0.97828
7	2.17310	3.6041	1.5842	2.1050	1.13786	1.33309	0.89016	0.97942
10^{2}	2.31180	3.8435	1.6136	2.1215	1.14582	1.335533	0.89347	0.980229
2	2.55945	4.2383	1.6513	2.1383	1.15545	1.338084	0.89741	0.981101
3	2.68505	4.4149	1.6648	2.1431	1.15874	1.338852	0.89874	0.981375
5	2.81785	4.5770	1.6760	2.1465	1.16141	1.339432	0.89982	0.981587
7	2.88883	4.6507	1.6809	2.1479	1.16257	1.339671	0.90028	0.981676
10^{3}	2.95016	4.7056	1.6847	2.1488	1.16343	1.339846	0.90062	0.981742
2	3.03360	4.7652	1.6891	2.1498	1.16445	1.340046	0.90103	0.981818
3	3.06508	4.7826	1.6906	2.1502	1.16479	1.340111	0.90117	0.981843
5	3.09188	4.7950	1.6918	2.15047	1.16507	1.340163		
7	3.10386	4.7998						
10^{4}	3.11305	4.8031	1.6927	2.15066	1.16527399	1.34020249	0.90136	0.9818783
2	3.12401							
3	3.12772							
5	3.13071							
7	3.13200							
10^{5}	3.13297	4.8093	1.6935	2.15082				
2	3.13410							
3	3.13448							
5	3.13478							
7	3.13491							
10^{6}	3.13501	4.8098						
2	3.13512							
3	3.13516							
5	3.13519							
7	3.13520							
10^{7}	3.13521591	4.8099						


6.2. Nappe confinée. Ecoulement axisymétrique. Soutirage à débit constant.
Drainance verticale. (Leaky aquifer)
no leakage
a/B = 0.1
a/B = 0.5
a/B = 1
a/B = 1.5
$\alpha = \frac{4Tt}{Sa^2}$
α
$\frac{s_0}{\left(\frac{-Q_0}{4\pi T}\right)}$
10
1
10^{-2}
10^{-1}
1
10
10^2
10^3

7. Principales fonctions rencontrées

a. dans les formules analytiques :

$$\operatorname{erf} x = \frac{2}{\sqrt{\pi}} \int_0^x e^{-x^2} dx \qquad\qquad \operatorname{erfc} x = 1 - \operatorname{erf} x = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-\xi^2} d\xi.$$

$$E_1(x) = \int_x^{\infty} \frac{e^{-t}}{t} dt \qquad\qquad Ei(x) = \int_{-\infty}^{x} \frac{e^{t}}{t} dt$$

J_0	Fonction	de	Bessel	d'ordre zéro	et de	première	espèce.
Y_0	"	"	"	" "	" "	seconde	"
J_1	"	"	"	" un	" "	première	"
Y_1	"	"	"	" "	" "	seconde	"
K_0	"	"	" modifiée	d'ordre zéro	et de	seconde	espèce.
K_1	"	"	" "	" un	" "	"	"

---

b. dans la méthode pseudovariationnelle.

	Par développement en série, on trouve la valeur de ces fonctions pour u petit.
$\varphi = 1 - e^{-2u} - 2ue^{-2u}$	$2u^2$
$\psi = -1 + e^{-2u} + 2u$	$2u^2$
$I_1 = 4u^3 - 7u^2 - 4ue^{-2u} + \frac{11}{2}u - \frac{5}{8}e^{-4u} + 2e^{-2u} - \frac{11}{8}$	$\frac{32}{15}u^5$
$I_2 = 2u^2e^{-2u} + 2u^2 + \frac{3}{4}ue^{-4u} + ue^{-2u} - \frac{13}{4}u + \frac{5}{8}e^{-4u} - 2e^{-2u} + \frac{11}{8}$	$\frac{6}{5}u^5$
$I_3 = -u^2e^{-4u} - \frac{3}{2}ue^{-4u} + 2ue^{-2u} + u - \frac{5}{8}e^{-4u} + 2e^{-2u} - \frac{11}{8}$	$\frac{4}{5}u^5$
$I_4 = 2u^2 - 2u - e^{-2u} + 1$	$\frac{4}{3}u^3$
$I_5 = ue^{-2u} + u + e^{-2u} - 1$	$\frac{2}{3}u^3$
$I_7 = u^3 - \frac{1}{2}u^2 - ue^{-2u} - \frac{1}{2}u - \frac{3}{4}e^{-2u} + \frac{3}{4}$	$\frac{5}{6}u^4$
$F = \frac{1}{u}(I_1\varphi - I_2\psi)$	$\frac{28}{15}u^6$
$F' = \frac{1}{u}(I_2\varphi - I_3\psi)$	$\frac{4}{5}u^6$

Notations.

$\dot{Q}$ débit (négatif si sortant) ; $Q = \int \dot{Q}\, dt$

q profondeur d'influence en écoulement parallèle.

R rayon d'influence en écoulement axisymétrique.

a rayon du puits.

$U_0 = \log \frac{R}{a}$, coordonnée généralisée, fonction du temps.

T transmissivité.

S coéfficient d'emmagasinement.

s rabattement de la nappe à la distance x ou r

s_0 " " " " " " " 0 ou a

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8. Bibliographie limitée à la justification des formules analytiques citées dans le texte.

$$\frac{\dot{Q}_o}{-\delta_o\sqrt{ST}} = \sqrt{\frac{1}{\pi t}} \quad \text{et} \quad \delta = \delta_o \, erfc\,(u) \qquad u^2 = \frac{Sx^2}{4Tt}$$

1.1.1. Ingersoll, L.R., Zobel, O.J. and Ingersoll, A.C. (1948). Heat conduction. New York. McGraw Hill.

$$\frac{\delta}{\left(-\frac{\dot{Q}_o}{T}\right)} = x\left[\frac{e^{-u^2}}{u\sqrt{\pi}} - erfc\,u\right] \qquad u^2 = \frac{Sx^2}{4Tt}$$

1.1.2. Ferris, J.G (1950) Quantitative method for determining ground-water characteristics for drainage design (Agr. Engineering 31.6. 285-291)

$$\frac{\dot{Q}_o}{-2\pi T\delta_o} = \frac{4\alpha}{\pi}\int_0^\infty x e^{-\alpha x^2}\left[\frac{\pi}{2} + tg^{-1}\frac{Y_o(x)}{J_o(x)}\right]dx \qquad \alpha = \frac{Tt}{Sa^2}$$

1.2.1. Jacob, C.E. and Lohman, S.W. (1952) Non-steady flow to a well of constant drawdown in a extensive aquifer (Am. Geoph. Union Trans. 33.4. 559-569).

$$\frac{\delta}{\left(-\frac{\dot{Q}_o}{4\pi T}\right)} = \int_0^\infty \frac{e^{-u}}{u}\,du \qquad u = \frac{Sr^2}{4Tt}$$

1.2.2. Theis, C.V. (1935) Relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using groud-water storage (Am. Geophys. Union Trans. p^t 2. 519-524)

$$\frac{\delta}{\left(-\frac{\dot{Q}_o}{4\pi T}\right)} = \frac{4}{\pi}\int^\infty_0 (1-e^{-\tau u^2})\,\frac{J_1(u)\,Y_o(\rho u) - Y_1(u)\,J_o(\rho u)}{u^2[J_1^2(u) + Y_1^2(u)]}\,du \qquad \tau = \frac{Tt}{Sa^2} \quad \rho = \frac{r}{a}$$

1.2.2. van Everdingen, A.F and Hurst, W (1949) (Trans. A.I.M.E. 186, 302)
ou Carslaw, H.S. and Jaeger, J.C. (1959) Conduction of heat in solids. Oxford. (formule 17 p. 338)

$$\frac{s_o}{\frac{-Q_o}{4\pi T}} = \frac{32 S^2}{\pi^2}\int_0^\infty \frac{1-e^{-\beta^2\frac{\alpha}{4}}}{[\beta J_o(\beta)-2SJ_1(\beta)]^2+[\beta Y_o(\beta)-2SY_1(\beta)]^2}\,d\beta. \qquad \alpha=\frac{4Tt}{Sa^2}$$

3. Papadopulos, I.S. and Cooper, H.H. (1967) Drawdown in a well of large diameter (Water Resour. Res. 3. 241-244)

$$\frac{s}{s_{oi}} = \frac{8S}{\pi^2}\int_0^\infty \frac{e^{-\beta\frac{u^2}{S}}}{u\{[uJ_o(u)-2SJ_1(u)]^2+[uY_o(u)-2SY_1(u)]^2\}}\,du \qquad \beta=\frac{Tt}{a^2}$$

4. Cooper, H.H., Bredehoeft, J.D. and Papadopulos, J.S. (1967) Response of a finite-diameter well to an instantaneous charge of water (Water Resour. Res. 3.1. 263-269)

$$\frac{-\dot{Q}_o}{2\pi T s_o} = \frac{a}{B}\,\frac{K_1(\frac{a}{B})}{K_o(\frac{a}{B})} + \frac{4}{\pi^2}\,e^{-\alpha(\frac{a}{B})^2}\int_0^\infty \frac{u\,e^{-\alpha u^2}}{[J_o^2(u)+Y_o^2(u)][u^2+\frac{a^2}{B^2}]}\,du \qquad \alpha=\frac{Tt}{Sa^2} \quad B^2=\frac{k'}{e'T}$$

6.1. Hantush, M.S. (1959) Non steady flow to flowing wells in leaky aquifers. (Journ. of Geophys. Res. 64.8. 1043-1052)

$$\frac{s}{\frac{-\dot{Q}_o}{4\pi T}} = 2K_o(2v) - \int_{\frac{r^2}{u}}^\infty \frac{1}{y}\,e^{-(1+\frac{v^2}{y})}\,dy \qquad v=\frac{r}{2}\sqrt{\frac{k'}{b'T}} \quad u=\frac{r^2 S}{4Tt}$$

6.2. Hantush, M.S. and Jacob, C.E. (1955) Non-steady radial flow in a infinite leaky aquifer. (Trans. Am. Geoph. Union. 36. 95-100)

Annexe 1.

Si la méthode est aussi performante, c'est qu'elle trouve sa justification dans quelque principe plus général. Effectivement et le lecteur qui souhaiterait approfondir la question lira avec profit :

Biot, M.A. (1970) Variational Principles in Heat Transfer. Oxford.
Biot, M.A. et Delmer, A. (1985) Variational-Lagrangian Analysis of Aquifers with Application to Artesian Wells (Water Resources Research. 21-2-249-255)

Annexe 2.

Correspondance entre les fonctions qui figurent dans la note de

Biot, M.A. et Delmer, A. (1985)		et dans ce travail.
$\mathcal{F}$	=	$\frac{1}{4}\varphi$
ϕ_1	=	$\frac{I_5}{2u_0^3 e^{-2u_0}}$
ϕ_2	=	$\frac{-\psi}{2u_0^2 e^{-2u_0}}$
B_1	=	$\frac{I_1}{16u_0^4}$
B_2	=	$\frac{I_2}{16u_0^3}$
F	=	$\frac{-I_1}{8u_0 I_4}$
F_1	=	$\frac{\psi}{4u_0}$
X	=	$-\frac{I_2}{8I_4}$
