Slow mass-movement and slope evolution in coherent and homogeneous rocks (*),

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RÉSUMÉ. — *L'intensité des déplacements visqueux et de ceux liés à des cycles expansion-contraction de la couche de débris est proportionnelle à la pente. Il en résulte l'évolution d'un versant plan vers un versant convexe avec diminution de l'angle de pente maximum. Ce résultat, obtenu en résolvant l'équation différentielle de* CULLING (9), *a été acquis par* YOUNG *par calcul numérique. Mais cet auteur le généralise à tous les types de déplacements lents de débris. Or, dans le cas où le versant est surtout soumis à des déplacements plastiques ou des déplacements liés à une rupture, une équation différentielle du type de la relation* (9) *ne peut être obtenue. Il existe en effet une pente critique sous laquelle le déclenchement ne peut se produire. Cette pente de déclenchement dépend du seuil de plasticité et de l'épaisseur de la couche de débris dans le cas de déplacements obéissant à la théorie classique de la plasticité et est égale au coefficient de frottement interne statique de la couche de débris dans le cas de déplacements liés à une rupture.*

L'application de la notion de bilan de matières (ablation, transport, accumulation) a permis de définir les divers types d'évolution et de concevoir les variations de pente observées sur les versants en fonction des variations des facteurs conditionnant les pentes de déclenchement et d'arrêt. Les corrélations obtenues par analyse d'un grand nombre de cas servent de confirmation aux idées exprimées. Il est bien évident que les conceptions de CULLING *restent applicables lorsque les déplacements liés à des cycles expansion-contraction du manteau de débris ou lorsque les déplacements visqueux jouent le rôle prédominant dans l'évolution des versants. De même d'ailleurs, les modèles de* SCHEIDEGGER *ne sont à appliquer que lorsque le weathering contrôle l'ablation sur des versants plus ou moins nus à pente forte.*

The concept of coherent rock presents no difficulties, but that of homogeneous rock does raise a problem of definition. In actual fact, no homogeneous coherent rock exists in nature. We shall therefore state that a slope is carved out of coherent and homogeneous rock when the rocks of which it is composed liberate, on weathering, products which are identical from the physical and chemical standpoints.

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We shall only examine those slopes that are covered with a continuous soil-mantle (¹) and we shall also presume that the weathering of the subjacent rock becomes negligible as soon as it is covered with a certain depth of soil, known as the critical depth.

The physical justification of such a connection between the intensity of weathering and the depth of the soil-mantle lies in the fact that the physical disintegration of the rock depends to a great extent on thermal variations and, in particular, on fluctuations around the freezing point of water. Now, FOURIER's law, which is applicable here, provides for the geometrical abatement of thermal variations in relation to depth. It is very difficult to calculate the critical depth in specific cases since this data integrates complex factors from two groups : the thermal properties of the mantle, and the climate at ground level. We know, for instance, that the abatement differs for oscillations of the same magnitude but differing in period : the annual oscillations go deeper than seasonal or diurnal variations. This raises the problem of the freezing limit of rocks in depth, which cannot be dealt with here.

The existence of a distinct limit between the subjacent rock and the soil-mantle justifies the expression « critical depth ». Indeed, a genuine discontinuity is noted between these two media which are very dissimilar from the standpoint of their physical properties. This observation would suggest that mechanical action predominates since, in such cases, chemical weathering does not lead to any profound change in the rock which would mask such contact.

In the following reflexions, it is also necessary that the local level of the bottom of the slope, represented by the position of the evacuation agency, should not undergo any substantial fluctuations from the vertical during the evolutionary process under consideration. This prerequisite is not an extremely restrictive one as the intensity of the shaping of the slope is, in most cases, far superior to the intensity of cutting or filling by the river.

In the evolution of slopes cut out of coherent, homogeneous rocks covered with a continuous soil-mantle, the slow mass-movement of particles, i.e. those in which inertial effects may be discounted, play a considerable part.

(¹) In this copy, the word soil is not used in a biological sense.

Among the ways in which material travel over the slopes, it is desirable to isolate mass-movements bound up with soil-mantle expansion and contraction cycles which need not detain us long. Thermal variations, hydrous fluctuations, alternating frost and thaw, all end in that result. In such cases, the intensity of the mass-movement (i.e. the quantity of material which, over a unit of time, passes through a vertical surface perpendicular to the vertical plane containing the line of greatest incline, of unit width and in other dimension equal to the depth of the non-weathered subjacent rock) is proportional to the slope (Culling, 1960, 1963; Scheidegger, 1960). Recent research (Young, 1963) has shown that, in many cases, these mass-movements play a secondary part, from the quantitative angle, in the evolution of slopes. More important in the transfer of masses are movements arising out of viscosity, plasticity or the phenomena of slow displacement due to shearing, the effect of which on the evolution of slopes we propose to demonstrate.

THE BASIC LAWS.

In a system of coordinates xy where the axis x follows the slope in a downwards direction and the axis y is perpendicular and directed towards the bottom, the fundamental relation can be expressed as :

$$\frac{-dv}{dy} = \frac{\tau - \tau_c}{\eta}, \tag{1}$$

where v represents the speed of the mass-movement, η the coefficient of viscosity, τ the tangential stress and τ_c the critical stress.

The tangential stress τ is the component of the weight following the grade (angle of slope : α) i.e. :

$$\tau = \mathrm{Sg} \sin \alpha \, y. \quad (\mathrm{Sg} = \text{specific weight}) \tag{2}$$

Let us first take $\tau_c = o$, the case of viscous mass-movements.

By placing (2) in (1) and integrating them, in the knowledge that for $y = a$, $v = o$, we obtain :

$$\mathrm{V} = \frac{\mathrm{Sg} \sin \alpha}{2\eta} (a^2 - y^2). \tag{3}$$

The mean speed is therefore :

$$V_{\text{mean}} = \frac{1}{a}\int_{o}^{a} \frac{\text{Sg} \sin \alpha}{2\eta}(a^2 - y^2)\, dy \tag{4}$$

or again

$$V_{\text{mean}} = \frac{\text{Sg} \sin \alpha \, a^2}{3\eta}. \tag{5}$$

The intensity of mass-movement I_d is therefore :

$$I_d = a V_{\text{mean}} = \frac{\text{S}ga^3}{3\eta} \sin \alpha. \tag{6}$$

For slopes inferior to 20°,

$$\sin \alpha \sim \text{tg}\,\alpha = \frac{\delta y}{\delta x}$$

and we find

$$I_d = \frac{\text{S}ga^3}{3\eta}\frac{\delta y}{\delta x}. \tag{7}$$

We know, moreover, that the intensity of the removal I_{ab} is equal to the variation rate of the intensity of mass-movement I_d according to x, i.e. :

$$I_{ab} = \frac{\delta I_d}{\delta x} = \frac{\text{S}ga^3}{3\eta}\frac{\delta^2 y}{\delta x^2}. \tag{8}$$

As the intensity of the removal I_{ab} is the variation in time of the ordinate y, by putting down

$$\frac{\text{S}ga^3}{3\eta} = \text{A},$$

we obtain

$$\frac{\delta y}{\delta t} = \text{A}\frac{\delta^2 y}{\delta x^2}. \tag{9}$$

This differential equation is identical to that of thermal conductibility. Its solution, similar to that obtained by CULLING (1960-1963) with respect to creep, shows that the maximum incline of the slope decreases in relation to time and that the normal profile is a convex curve in cases where the materials are evacuated at the supposeddly fixed base of the slope.

Let us now state $\tau_c = k$, TRESCA's criterion of plastic mass-movement.

By placing (2) in (1) and integrating them, knowing that for $y = a$, $v = o$, we obtain :

$$\mathrm{V} = \frac{\mathrm{S}g \sin \alpha}{2\eta} (a^2 - y^2) + \frac{k}{\eta} (y - a). \tag{10}$$

The mean speed is therefore :

$$\mathrm{V}_{\mathrm{mean}} = \frac{1}{a} \int_o^a \left[\frac{\mathrm{S}g \sin \alpha}{2\eta} (a^2 - y^2) + \frac{k}{\eta} (y - a) \right] dy \tag{11}$$

or again :

$$\mathrm{V}_{\mathrm{mean}} = \frac{1}{a} \left(\frac{\mathrm{S}g \sin a^3}{3\eta} - \frac{ka^2}{2\eta} \right), \tag{12}$$

and the intensity of mass-movement I_a is :

$$\mathrm{I}_a = a\mathrm{V}_{\mathrm{mean}} = \frac{a^2}{6\eta} (2\mathrm{S}g \sin \alpha\, a - 3k). \tag{13}$$

For mass-movement to occur, it is necessary that

$$\mathrm{S}g \sin \alpha a = \beta k, \tag{14}$$

where β is a constant.

Relation (14) is a fundamental one; it indicates the factors which condition plastic mass-movement, according to TRESCA's criterion, on a given slope. In the case of the plastic flow of glaciers, NYE (1951, *in* SCHEIDEGGER, 1960) obtains a similar solution and calculates the components of the glacier speeds according to x and y on the basis of that result. These data are of lesser interest to us here, for we are more concerned with the concept of the balance profile of the slope under the action of the various modes of soil travel defined at the outset rather than with their evolutionary rate.

Relation (14) can also be expressed for slopes inferior to 20° where $\sin \alpha \sim \operatorname{tg} \alpha$:

$$\operatorname{tg} \alpha = P = \frac{Sga}{\beta k} \cdot \tag{15}$$

The thicker the soil mantle, the slighter will be the launching slope. The higher the plasticity threshold k, the greater will be the launching slope. At the limit, when $k = o$, the launching slope is nil; we are now in the sphere of viscosity which has already been examined above.

Furthermore, there is a critical value of depth a if mass-movement is to occur on a given slope $\operatorname{tg} \alpha$.

Other types of slow mass-movements obey the criterion of Coulomb which may be expressed as follows :

$$\tau_c = KSg \cos \alpha y + C, \tag{16}$$

where K and C are two constants known respectively as the coefficient of inner friction and the coefficient of cohesion. It should be noted here that a mass-movement guided by the relation (16) is not necessarily a rapid one : everything depends on the distribution of principal stresses. Starting from conditions of static equilibrium where the inertial effects have been disregarded it may be demonstrated that, if $C = o$, the launching slope is given by the formula :

$$K = \operatorname{tg} \alpha. \tag{17}$$

This slope is therefore equal to the coefficient of inner friction of the material, which is usually shown as $\operatorname{tg} \emptyset$ ($= K$). Cohesion C comes from the link that exists between the particles as a result of the presence of a film of water surrounding each grain. We see that if $K = \operatorname{tg} \emptyset = o$ in relation (16), the expression of critical tension becomes analogous to Tresca's criterion. This means that the theory of plasticity is applicable on condition that the coefficient of inner friction is nil. Clay and argillaceous silt in practice evidence this characteristic when they are thoroughly wet. If, on the other hand, the soil mantle is composed mainly of sand or gravel particles, or particles that are still bigger, i.e. with a negligible cohesion, then the slow mass-movement will be regulated by law (17) which states that, on balance, the slope is equal to the coefficient of inner friction. In an intermediate position, cohesion may be responsible for the existence of slopes steeper than those corresponding to the coefficient of inner friction.

While it is responsible for a decrease in cohesion, the rise in the water content also causes a decrease in the coefficient of inner friction, the water playing the part of a lubricant. It therefore becomes difficult, in the case of a soil mantle in which $\mathrm{tg}\ \emptyset \gg o$ and $C \gg o$, to ascertain the relative importance of the two factors in the correlative abatement of the slope. The coefficient of inner friction also depends on the index of voids which is the ratio of the volume of voids to the volume of solid matter. The relation given by TERZAGHI (1961) is the following:

$$\mathrm{tg}\ \emptyset = 0.55 / \varepsilon \tag{18}$$

where ε is the index of voids.

Now, this index depends on the shape of the particles, the degree of uniformity in the granulometric spectrum, and pressure, i.e. settling. Finally, it is desirable to introduce the concepts of static coefficient of inner friction ($\mathrm{tg}\ \emptyset$) and kinetic coefficient of inner friction ($\mathrm{tg}\ \varphi$). When the speed of a mass of particles goes from nil to a value of any kind (however small) the coefficient of inner friction decreases in value by passing from $\mathrm{tg}\ \emptyset$ to $\mathrm{tg}\ \varphi$. Because of this, the condition of critical balance which was formerly $\mathrm{tg}\ \emptyset = \mathrm{tg}\ \alpha$, now becomes $\mathrm{tg}\ \varphi = \mathrm{tg}\ \alpha$. This means a balance slope that is slighter at the point of stopping than at the point of launching. Thus, by rejoining here the ideas advanced by SHARPE (1938) and STRAHLER (1950), it is possible to define a threshold of launching which is higher than a threshold of stopping. If the speed goes from a value V_1 to a value V_2 (V_2 being far greater than V_1), $\mathrm{tg}\ \varphi_1$ can be slightly higher than $\mathrm{tg}\ \varphi_2$, but this does not affect our theory since we are disregarding the inertial effects. Whatever the type of mass-movement may be, two situations may arise : RANKINE's active state and passive state, corresponding to extensive and compressive mass-movements. PRANDTL has calculated the shapes of the slip-lines, which are sections of cycloids descending in a downslope direction, thus permitting the starting in the first instance, or rising in an upslope direction, thus permitting the overthrust in the second instance. SCHEIDEGGER (1960) has taken NYE's figures and presented them once again in his treatise.

COEFFICIENT OF INNER FRICTION AND EVOLUTION OF SLOPES.

If $I_{ab} = \frac{\delta I_d}{\delta x} > o$ for a specific sector of slope, a removal (or denudation) sector will be referred to. The slope must at least be equal to that which corresponds to the threshold of launching, i.e. to the static coefficient of inner friction.

If $\frac{\delta I_d}{\delta x} = o$ for a specific sector of slope, the intensity of mass-movement will be constant and the sector under consideration will be known as the sector of transportation.

If $\frac{\delta I_d}{\delta x} < o$ for a specific sector of slope, this will be known as a sector of deposition. The incline will be relative to the threshold of stoppage, i.e. the kinetic coefficient of inner friction. Consequently, a removal sector shows steeper inclines than a sector of transportation and, a fortiori, than a sector of deposition. The maximum incline of the slope (¹) is, therefore, inevitably part of the removal sector and the basal concavity of the profile, if it exists, will be relative either to the removal or the transportation, or again, the accumulation, which makes it possible to grasp the opposition between convex-concave slopes where the soil depth is noticeably the same throughout the profile (removal or removal-transportation) and convex-concave slopes evidencing a deepening of the soil at their bases (accumulation).

We may therefore define three fundamental types of slope profiles :

a) a removal profile;

b) a removal-transportation profile;

(¹) By maximum incline of slope, we are referring to the mean maximum incline = the mean incline of the steepest sector over a difference in level of at least 10 metres. The latter is suffcently large to escape the influence of accidental factors, yet has no great significance in a general survey of a slope (concept of scale).

c) a removal-transportation-accumulation profile with a transport sector that is more or less developed and may even be non-existant.

THE REMOVAL PROFILE.

Three questions arise in regard to this profile :

a) Why is it that, in some cases, the evolution tends to level off while, in others, the retreat of the slope is effected at constant maximum incline ?

b) What is the explanation of summit convexities with a wide curvature radius ?

c) Can a basal concavity exist in the case of a removal profile ?

A removal profile may evolve parallel to itself if the coefficient of inner friction of the soil mantle undergoes no variations with the passage of time. The abatement of the maximum incline of the slope is the result of a decrease in the coefficient of inner friction over a period of time. If the soil mantle is not to disappear during the evolutionary process, the weathering rate of the subjacent rock I_{ar} (the quantity of materail supplied by weathering over a period of time *dt*) must be equal to the removal rate I_{ab} (quantity of material which disappeared from the sector concerned over the period of time *dt*). This equality can be achieved for differing values of the weathering rate of the soil mantle I_{ac} (quantity of material passing from a given state of weathering to another over the period of time *dt*).

If $I_{ac} = I_{ab} = I_{ar}$, the soil mantle retains its characteristics in time and the retreat can only be « parallel » (of constant maximum incline). If $I_{ac} > I_{ab} = I_{ar}$, the soil mantle will undergo development by weathering which, as we shall demonstrate later on, ends in a reduction of the coefficient of inner friction, i.e. an abatement of the maximum incline of the slope. An even slope with an incline of Ø can thus decrease by *d* Ø following a differentiation and evolution of the soil mantle. If a removal profile is to retreat at constant maximum incline during the evolutionary process, then the agency evacuating the soil must migrate in the same direction during the period of time under review.

Now let us consider possible variations of the coefficient of inner friction. The intense weathering of the soil mantle leads to a decrease in the calibre of the materials of which it is composed. Now, there is no direct connection between the coefficient of inner friction and the mode of the granulometric spectrum. Consequently, how are we to understand the influence which the weathering of the soil mantle exerts on the coefficient of inner friction ? The result of the decrease in the calibre of the materials is a heightened water retention capacity and a lower filtration rate. An increase in the water content may occur, and we know that this means a decrease in the coefficient of inner friction. Furthermore, weathering of the soil may lead to the production of less angular fragments, which brings about a reduced tg Ø. Finally, a rise in the index of voids may also have an effect. Here, the question is one of a variation in the coefficient of inner friction in time. Cannot similar fluctuations also occur in space ?

It often happens that a difference is revealed between the edge of the plateau and the slope itself. The soil mantle is more highly evolved on the edge of the plateau than it is on the slope. Later weathering stages become apparent since the soil has in turn been subjected to weathering.

A more probable increase in the water content and a variation in the compactness of the soil will explain a variation in the coefficient of inner friction as one moves from the slope towards the plateau. If we know that we may associate incline and coefficient of inner friction, then the foregoing comments will satisfactorily explain the summit convexity. If the static coefficient of inner friction remains fairly constant towards the top, the summit convexity will be only slightly developped and the straight cross-section will be more extensive. This explanation is valid for a profile and for a removal sector as well. The effect of variations in space may be combined with that of variations in time. If so, the summit convexity and the evolution towards a levelling off will both appear in conjunction.

Are there any basal concavities in a removal profile ? A decrease in the static coefficient of inner friction towards the bottom of the slope could make it possible to conceive of such a concavity. But since many convex-concave slopes with a continuous soil mantle show no granulometric differentiation, only a rise in the water content would explain the decrease in the static coefficient on inner friction. But then the observation shows that a very substantial proportion of fine particles is needed if the variations in the coefficient of inner friction as a result of the change from dry to wet are to be as large as the recorded variation of incline. And, in this case, the mass-movement, which is impossible when the water content of the mantle is low owing to a particularly high degree of cohesion, occurs according to the classic theory of plasticity (coefficient of inner friction nil). Now, that problem is not under consideration at this time. A variation in the cohesion (C) could also explain the presence of removal concavities. But the frequent presence of very handsome concavities with a very gravelly soil mantle (where $C = 0$) provides a major objection to this reasoning. Altogether, these facts consequently represent a limit to the developement of removal concavities and also to the exclusive part played by the water content in the genesis of such formations.

THE REMOVAL-TRANSPORTATION PROFILE.

We shall presume that the initial state is composed of a slope with a rectilinear profile covered with a homogeneous soil mantle and with an incline Ø (fig. 1). The depth of the mantle is identical throughout the length of the profile. The intensity of weathering of the underlying rock is in inverse ratio to the depth of the soil mantle. It becomes negligible when the latter attains a critical value. A stream acts as the evacuation agency. Its position does not change with the passage of time.

These initial conditions are indicated on figure 1 by a line marked 1 which corresponds to the position of the underlying rock, and another line marked A representing the ground surface. The stream located at the foot of the slope is perpendicular to the level of the sheet. If a mass-movement occurs, even an infinitely small one, the ground surface cannot undergo a strictly parallel retreat because the soil would no longer be eliminated by the river, in the light of the distance in relation to the evacuation agency. The new situation B of the ground surface will be as follows : profile parallel to A in the upper part, and a less inclined junction sector in the lower part. The grade of this sector is sufficient to allow of the evacuation of the soil : thus we have here a sector of transportation, the grade of which is very slightly superior to the kinetic coefficient of inner friction, i.e. superior to the threshold of stoppage by an infinitely small margin. The slope tg Ø, moreover, corresponds in the same way, by an infinitely small

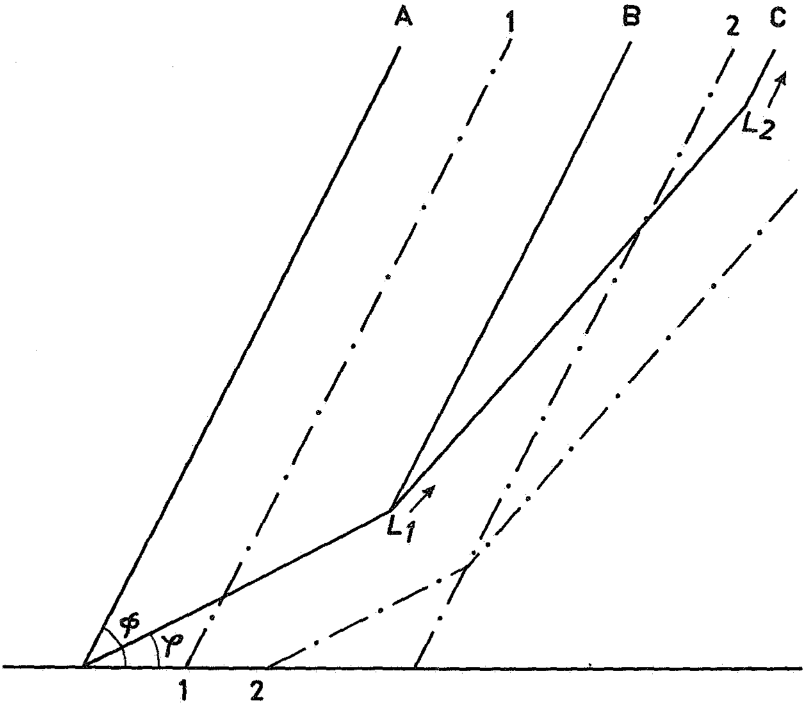

FIG. 1.

margin, to the static coefficient of inner friction. The difference between the static and kinetic coefficients of inner friction, i.e. between the threshold of launching and the threshold of stoppage, enables us to explain the maximum and minimum inclines limiting the sector of transportation. But, how, then, is one to understand the concave nature of that sector, which develops a series of intermediate slopes between the maximum incline of the slope in the removal sector and that which corresponds to the threshold of stoppage ? These intermediate, or transitional, inclines are explained by the increasingly marked compression affecting the soil mantle as one approaches the bottom of the slope. Now, we have seen that this compressive state reveals slip surfaces permitting the overthrust of the materials as far as the stoppage grade. As the retreat continues, the style of the slip becomes less and less compressive since the height of the removal sector decreases. The possibilities of overthrust along the slip surfaces grow less, which leads to the formation of stronger and stronger corresponding junction grades (position C of fig. 1). If the removal sector evolves towards a summit convexity, the explanation of which has been given above, the profile as a whole will be convex-rectilinear-concave or convex-concave with a constant soil depth. It will be a removal-transport profile with a concave sector of transportation.

In figure 1, we also see that point L separating the removal sector from the sector of transportation — this being the inflection point of a convex-concave profile — migrates upwards during the evolutionary process insofar as the maximum incline of the slope remains constant.

Now let us see how the evolution towards levelling off takes place. As a point of departure, we are considering a slope with a well-developed concavity in the sector of transportation. When the maximum incline of the slope goes from an incline $\emptyset$ to an incline $\emptyset - d\,\emptyset$ (with $d\,\emptyset > 0$) following a reduction in the static coefficient of inner friction, the sector of transportation will decrease to the advantage of the removal sector (fig. 2). The new profile will be given by line B in figure 2,

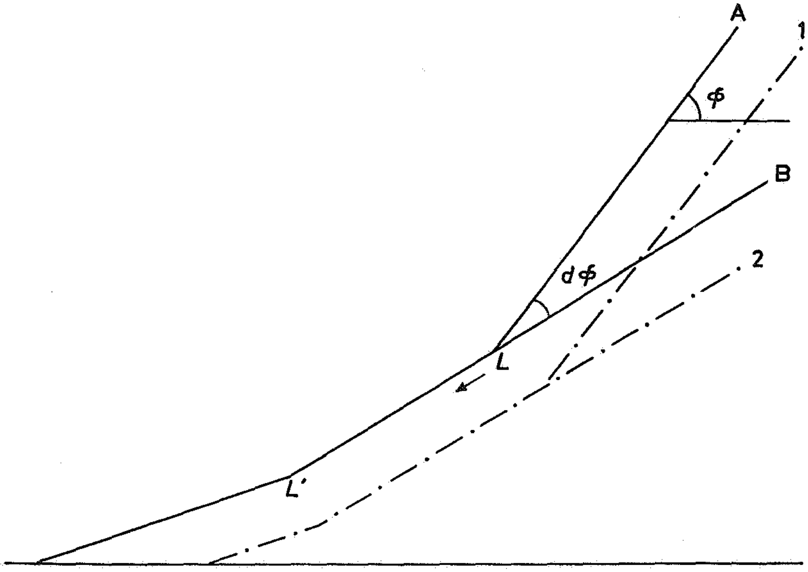

FIG. 2.

point L will migrate towards the bottom at L′. Consequently, there are two opposite migration trends evidenced by the point of inflection or the point separating the removal sector from the sector of transportation : a migration towards the top following the development of the sector of transportation when the straight part of the removal sector retreats parallel to itself (tg Ø = constant) and a migration towards the bottom as a result of levelling off (tg Ø diminishing).

This shows that it is pointless to measure the importance of a sector of transportation in natural conditions without taking special precautions : a comparison will only be possible if the maximum inclines of the slopes under consideration are the same but, even so, the existing situation will largely depend on previous situations.

THE REMOVAL-TRANSPORTATION-ACCUMULATION PROFILE.

Let us begin by presuming a removal-accumulation profile, i.e. one in which the sector of transportation is reduced to a dot. For one reason or another, at one point the agency evacuating the soil no longer skirts the bottom of the slope. This case may arise, for instance, with a convex bank owing to a centrifugal displacement of the river. An accumulation of soil is consequently possible. The underlying rock being protected by a certain depth of soil, the progressive accumulation confers a convex profile on the underlying rock. The upper part of the profile tends to arrange itself parallel to the surface of the ground (fig. 3). Richter's homologue of the incline (Richter, 1901; Lehmann, 1933) in the case of a slope with a continuous soil mantle is the incline of the rock face at the upper limit of the accumulation sector. Its physical significance is therefore clarified since it corresponds to the kinetic coefficient of inner friction of the soil mantle. However, should the accumulation be subsequent to the development of an important sector of transportation, the profile of the underlying rock may be concave at the lowest extremity of the slope. This part of the profile corresponds simply to the smothering of the former sector of transportation through the piling up of soil (fig. 4).

Accumulation is usually evident in the following cases :

1° the withdrawal of the evacuation agency from the base of the slope;

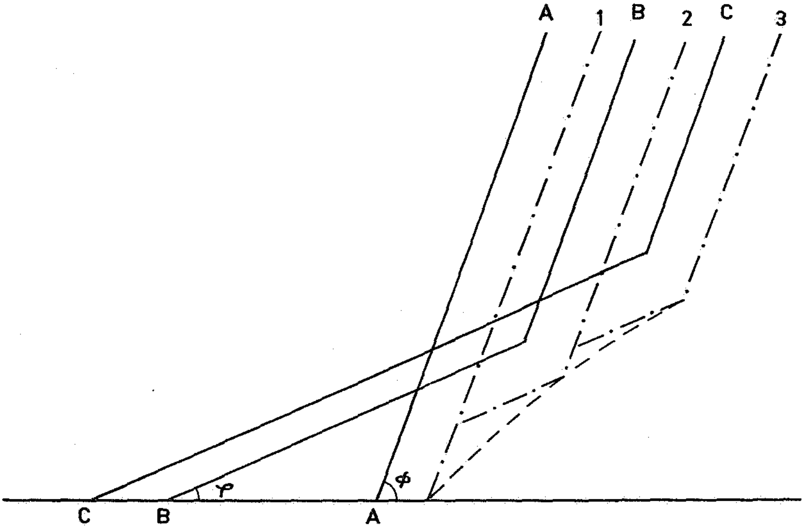

FIG. 3.

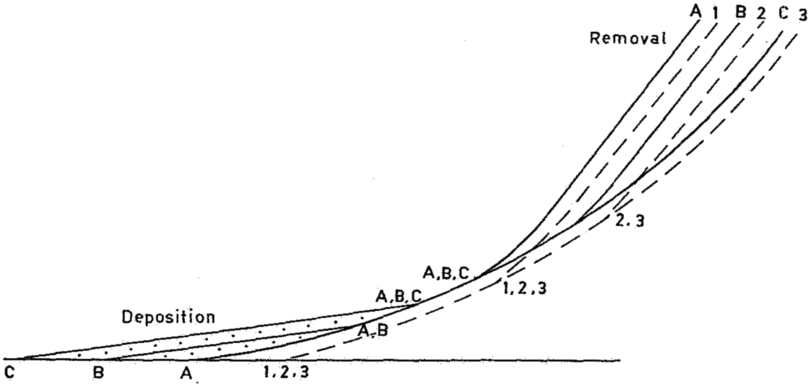

FIG. 4.

2° the impossibility, for the evacuation agency, of carrying the soil coming from the evolution of the slope because of its over-large calibre : accumulation due to incompetence;

3° the impossibility, for the evacuation agency, of carrying the soil coming from the evolution of the slope because of the over-large quantity delivered during the period of time under consideration : accumulation due to incapacity;

4° when two or three cases occur in conjunction.

Accumulation can very well occur locally even if the river tends to excavate slightly. Unless it is generalised, it has no need of a stream still in the filling stage. Accumulation on the convex bank following a migration of the axis of the bend is a demonstrative example of this fact. Why can we refer to an accumulation concavity ? We have previously noted that the kinetic coefficient of inner friction determined the threshold of stoppage. In theory, therefore, below the sector of transportation the inferior limit of which is provided by the threshold of stoppage, the incline should no longer vary : the concavity of the transport sector should be followed by a rectilinear accumulation slope. The grade reduction in the accumulation sector itself is due to a selective stoppage of particle masses that have different water contents at the time of their arrival at the bottom of the slope. In a given climate and for a specific mantle, there are maximum and minimum water content values that are periodically reached. The corresponding threshold of stoppage, bound up with the concept of the kinetic coefficient of inner friction, will respectively have lower and higher values, which will explain the presence of an accumulation concavity.

THE EVOLUTION OF SLOPES THROUGH PLASTIC MASS-MOVEMENTS.

When the soil mantle contains enough fine particles ($<10\ \mu$), its high degree of cohesion prevents any mass-movement when the water content is low. If the material is thoroughly wet, the coefficient of inner friction falls to nil. The classic theory of plasticity becomes applicable, therefore, since TRESCA's criterion has been met.

Under such conditions, any mass-movement requires that the following equation be obtained :

$$\operatorname{tg} \alpha = \frac{\beta k}{Sga} \cdot \qquad \text{(cf. relation 15)}$$

We know that the mass-movement may occur either in the active state or the passive state according to RANKINE's definition. The soil mantle always adopts the type of flow that maintains the critical depth given by relation (15). Consequently, a convex zone promotes an active flow, and a concave zone promotes a passive flow.

Let us suppose that the critical depth is attained in the various cases. Depending on the types of clay, the slope necessary for the mass-movement of the materials subjected to plastic deformation varies from 1 to 5°. Because of this, the concepts of removal, transportation or accumulation sectors lose a major part of their validity. A slope which evolves according to the process we are now describing tends to level off. Its incline depends at any given moment on the intensity and duration of the effects of this process. No retreat in parallel to itself is possible except if the evacuation agency undergoes a strong lateral migration in the same direction. As this type of mass-movement depends on the water content of the soil mantle, the latter must not reveal too low a filtration rate but must retain a certain degree of porosity. There is, therefore, a certain discrepancy between the run-off and the plastic mass-movement.

A CONFIRMATION OF THE FIELD.

The Meuse area of Lorraine, in both French and Belgian territory, presents a range of calcareous rocks which reacted to the cold climates of the Quaternary period according to their physical properties. Their weathering has led to the creation of various types of soil mantles. In this context, it is possible to select lithologically homogeneous slopes with a continuous soil mantle, slight effects of the run-off, and similar development. In view of the stability of the basic level of the Meuse region during the recent Quaternary period (SOUCHEZ, 1963; TRICART, 1952), this region stands revealed as a choice field for use as an « experimental » confirmation of our theory. We have studied elsewhere (SOUCHEZ, 1963 *a*, 1963 *b*) the genesis of soil mantles. Owing to their particularities which are explained by weathering, the latter have different physical properties which will influence the way in which mass-movement takes place and, consequently, the evolution of the entire slope (JANN, 1963; Starkel, 1964). Let us therefore examine the connection between the properties of the soil mantle and the features of the slopes surveyed.

THE CONNECTIONS BETWEEN THE MEAN MAXIMUM INCLINE AND THE PROPERTIES OF THE SOIL MANTLE.

Following the example set by Tricart and Muslin (1951), we shall define the mean maximum incline of a slope as the mean incline of the steepest sector over a difference in level of about 10 m. The latter is large enough to escape the influence of accidental factors, while it has little significance in a general study of the slope (concept of scale).

We have often had occasion to note that, for a simple lithological formation, the mean maximum inclines of slopes of the same size are closely grouped around a mean value.

Once more, this fact merely serves to confirm the remarkable statistical correlation pointed up by Strahler (1950). It can be clarified in terms of physics through the relationship between the static coefficient of inner friction and the balance slope.

It is now necessary to demonstrate that good correlations exist between the properties of the soil mantle, which influence the value of the static coefficient of inner friction, and the mean maximum inclines. Beforehand, let us comment that Strahler's correlation is less well verified for slopes covered with a soil mantle containing a great many fine particles. In this case, the mean maximum incline of the slope has no precise physical significance since it only depends on the rapidity with which the slope evolues due to weathering and slow mass-movements bound up with plastic deformation. It is therefore normal that Strahler's correlation should be less clearly verified. In other cases, however, the mean maximum incline is very significant indeed because it is bound up with the concept of the coefficient of inner friction. A series of correlations will confirm this theory.

The granulometric spectrum of a soil mantle has a very important but complex incidence on the coefficient of inner friction. We shall characterise it by means of three separate data : the mode, the percentage of particles inferior to 100 microns, and the sorting index of Trask.

The mode or most frequent grain constitues a characteristic value of the deposit and is far less affected by subsequent disturbances than the median. Let us eliminate the easily recognised case where there is an enrichment of fine particles by addition of eolian silt. The weathering of the deposit by chemical corrosion must, however, receive our closest attention. It generally leads to the constitution of a residue that is very rich in fine particles less than 10 microns in diameter. We must therefore bear this influence in mind in our choice of the final dimensions which are to characterise the fine particles. If we select the percentage of fine particles with a diameter of less than 10 microns in characterising a soil mantle, we run the risk of making an error of interpretation because a certain number of particles of less than 10 μ are not necessarily contemporaneous with the formation of the slope, but may be the result of a recent development due to chemical weathering, the modelling being practically stable by the vegetation at the present time. If we choose the percentage of fine particles with a diameter of less than 100 microns, the mistake tends to be ruled out. This percentage of fine particles will test the probable extent to which such a mantle will acquire a high water content since we have found good correlations between it and the retention capacity.

Let us consider the overall relationships between the mean maximum incline and the properties of the soil mantle.

No clear relationship arises when comparing the mean maximum inclines with the modes of the granulometric spectra.

With a similar development of slopes, almost identical maximum inclines of 25° occur even though the modes are 200 microns and 6 mm. respectively. In the same way, modes that are very similar (200 μ) are accompanied by different maximum inclines (27 and 18°). This relative independence between the maximum incline of a slope formed by slow mass-movements of soil and the mode of the mantle which covers it is quite conceivable since the index of voids, linked to the coefficient of inner friction by TERZAGHI's relation $\operatorname{tg} \emptyset = 0.55/\varepsilon$, is independent of the size of the particles.

With the ratio m, $\left[\dfrac{\text{p, percentage of fine particles} < 100\ \mu}{(\text{S, Trask sorting index}) \times (\text{o, constant})}\right]$, the relationships are clearer (fig. 5). We see that the higher the ratio m, the weaker the corresponding incline. The connection between the percentage of particles inferior to 100 microns and

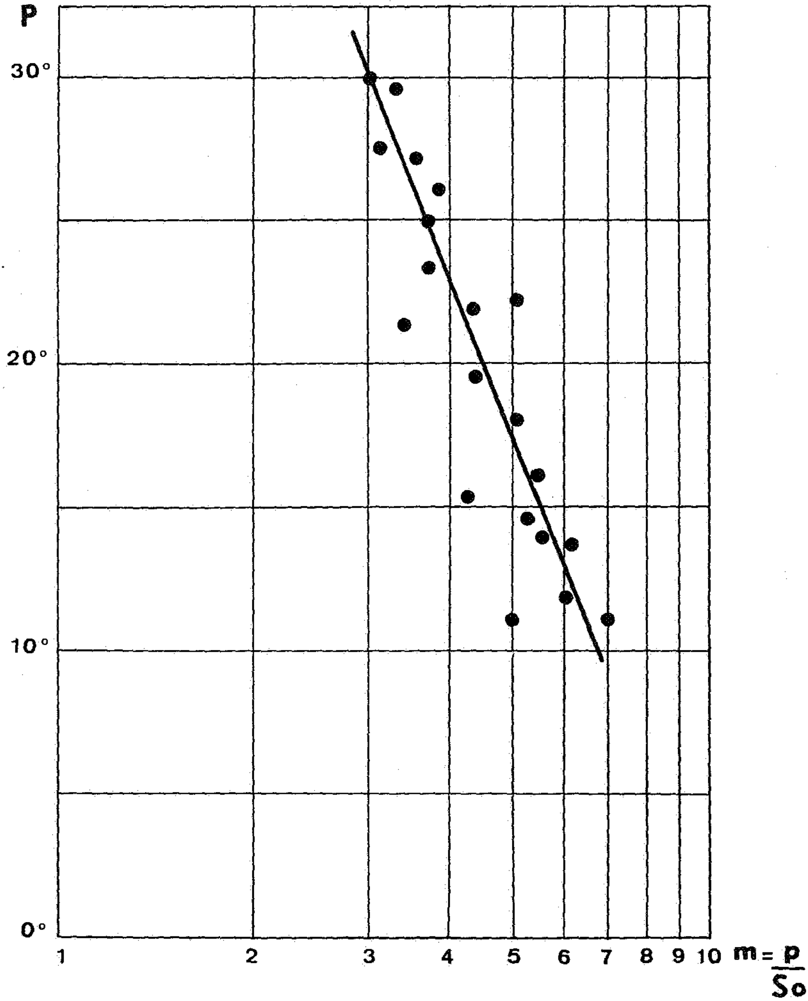

FIG. 5.

the mean maximum incline may appear curious, in the sense that the index of voids is independant of the size of the particles. The fact may, however, be explained by the following reason : when a soil mantle contains more fine particles, its properties with regard to water vary. Indeed, the retention capacity increases when the percentage of particles of equivalent size inferior to 100 microns is higher. Similary, the filtration rate, which defines the porosity of the material, decreases. Now, the coefficient of inner friction, while it depends on the index of voids, also fluctuates in relation to the water content. Acting as a lubricant, the water decreases the value of tg Ø.

We must also regard the effect of the sorting index. The greater the uniformity of the material and the lower the index, the lower will be the value of the coefficient of inner friction since the index of voids is correspondingly higher. These definite relationships between the properties of the mantle which condition the value of the static coefficient of inner friction, and the mean maximum inclines are already an « experimental » confirmation of our theory which sheds light on the physical significance of these inclines. The granulometric spectrum of the mantle, a synthetic factor, plays a complex part that is extremely important but very difficult to define. In point of fact, there is no simple and direct relationship between the mode and the maximum incline.

RELATIONSHIPS BETWEEN THE MAXIMUM ACCUMULATION SLOPE AND THE PROPERTIES OF THE MANTLE.

Let us give the name « maximum accumulation slope » to that which corresponds, at the limit of the transportation and accumulation sectors, to the kinetic coefficient of inner friction. If the mass-movements are slow or fast, the value of the kinetic coefficient of inner friction is very different from that of the static coefficient of inner friction since it corresponds to the minimum incline needed to maintain the movement once it has begun. The value of this coefficient may vary slightly with the speed of the movement, but the variation is not on the same scale — it is slighter — as that which corresponds to the passage from an initial speed of nil to a given speed, however slow it may be.

To discern beyond all argument a deepening of the mantle in the basal concavity, it is almost necessary for a quarry to cut into the slope of the right spot. We have found such zones of accumulation and these show that the higher the mean maximum incline of the slope, the higher the maximum accumulation slope tends to be. This observation can easily be explained since the maximum accumulation slope, just like the mean maximum incline, depends on the percentage of fine particles.

THE PROBLEM OF SO-CALLED TRANSPORTATION CONCAVITIES.

Let us show that, in the cases under review, the concavities which do not present any deepening of the soil mantle cannot be removal concavities. In other words, they cannot exclusively be the outcome either of a grain size variation or of an increase in the water content towards the bottom of the slope.

Various authors have stressed the importance of a decrease in the grain size towards the bottom of the slope in order to explain the basal concavity. If we take this granulometric decrease — a rather vague term — to mean a variation in the mode, we cannot confirm this point of view. In actual fact, the measurements we have carried out do not enable us to discern a systematic decrease in the mode (the grain which is most frequently found) towards the bottom of the slope. As for the median, we have pointed out its lack of real significance. It should, moreover, be noted that we have shown the relative independence of the mode concept with regard to that of slope.

The differences observed in the modes are on the same scale as the precision of the measurements; consequently they are of no significance. Are not the basal concavities explained by a larger percentage of fine particles with a diameter of less than 100 microns ? As with the modes, the differences observed are on the same scale as the precision of the measurements.

The influence of the water content must also be reckoned with (Birot, 1949). If there can be no valid measurement of the mean coefficient of inner friction of the soil mantle in the laboratory — analysis of a sample will often provide a very rough approximation — it is nevertheless possible to estimate the maximum variation of the balance slope by proceeding from the dry to the saturated state. In the case of a type of sand similar to that covering the slopes hewn from Sinemurian calcareous cement sandstone in Belgian Lorraine, values in the vicinity of 3 to 5° are obtained. This value of only 3 to 5° is greatly inferior to the slope variation observed, which is around 15°. In the case of Sinemurian calcareous sandstone, this fact limits the possibility of removal concavities. Therefore the basal concavity would essentially seem to be a transportation one, but it is obvious that the influence of the rising water content towards the bottom can be added to that of the slope variation resulting from the difference between the static coefficient of inner friction and the kinetic coefficient of inner friction.

In the case of Bathonian cretaceous limestone and the various calcareous formations of the Meuse Heights, the situation is analogous : the value measured in the laboratory rises with the percentage of particles less than 100 microns in diameter but always remains inferior to the slope variations observed. To obtain commensurate results, the mantle should be of silt or argillaceous silt, but at this point plastic deformations play a paramount part. Now, it is not possible to apply the same laws to the shaping of slopes with a gravelly soil mantle as to those with a mantle rich in fine particles.

One final observation seems to be of interest. It is well known that a deposit linked with a mass-movement presents not only a majority of thin slabs parallel to the slope but also a high percentage of relevant elements. These relevant elements would seem to materialise the overthrust slip surfaces of RANKINE's compressive or passive state. Now, in the case of Sinemurian calcareous sandstone which may be taken as an example, the percentage of relevant elements (thin slabs) becomes very large in the concavities and attains value sof 72 % to 81 %. Is this not another fact that comes to the support of our interpretation ?

THE PROBLEM OF SUMMIT CONVEXITIES.

The summit convexity is particularly well developped in Sinemurian calcareous sandstone, Bathonian cretaceous limestone and the coralline limestone of the Meuse Heights.

If we examine the composition of the soil mantle on the edge of the plateau and compare this data with the grain size analysis of the deposits on the slopes, we observe a slight increase (max. 2 %) in the percentage of fine particles from the slope towards the edge of the plateau. We have already seen, with regard to the significance of the maximum incline, the incidence of this fact on the coefficient of inner friction.

The very slight differentiation (0,5 %) between the edge of the plateau and the slope in the case of Sinemurian calcareous sandstone allows the supposition that variations in the compactness of the soil play the essential part in this case. According to TERZAGHI's data, we do in fact know that in fine sand, the porosity of which varies from 48 to 41 %, the angle of inner friction rises from 28 to 35°. Slight variations in porosity may therefore have important consequences. We have been unable to confirm the part played by the index of voids in view of the uncertain character of any measurement of the soil porosity on the spot, and in view of the fact, with the development of the forest, conditions are certainly no longer what they were when the slopes under review were in course of formation.

CONCLUSION.

The intensity of viscous mass-movements and those bound up with expansion-contraction cycles in the soil mantle is proportional to the slope. The result is an evolution from an uniform slope towards a convex slope with a decrease in the maximum angle of incline. This result, similar to that obtained by solving CULLING's differential equation (9), has also been partially obtained by YOUNG (1963) for the creep through numerical computation. But this author has extended it to all types of slow mass-movements of soil. Now, in cases where the slope is mainly subjected to plastic mass-movements or those connected with shearing failure, a differential equation of the relation (9) type cannot be obtained. There is in a fact a critical slope below which the launching cannot occur. This launching slope depends on the plasticity threshold and the depth of the soil mantle in the case of mass-movements which obey the classic theory of plasticity, and is equal to the static coefficient of inner friction of the soil mantle in the case of mass-movements connected with shearing failure.

The application of the concept of an evaluation of contents (removal, transport, accumulation) has made it possible to define the various types of evolution and to form an idea of the slopes variations observed on the slopes in relation to the variations of those factors which condition the launching and stoppage inclines. The correlations obtained by analysing a great many cases serve to confirm the ideas put forward. It is quite evident that CULLING's concepts (1960-1963) remain applicable when mass-movements are linked with expansion-contraction cycles in the soil mantle, or when viscous mass-movements play the predominant part in the evolution of the slopes. In the same way, moreover, SCHEIDEGGER's constructions are only applicable when weathering regulates removal on more or less denuded slopes with a steep incline.

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